DELEGATION THEORY AND THE DELEGATION OF TARIFF-Negotiation AUTHORITY

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Abstract

This paper explores the relationship between the theory of optimal delegation and the delegation of tariff-negotiation authority under U.S. legislation following the Reciprocal Trade Agreements Act (RTAA) of 1934. In the first part of the paper, we specify general welfare functions for the principal and agent into which a bias parameter enters in a separable fashion. For the delegation problems with and without money burning, we then apply results from Amador and Bagwell (2013) and provide general conditions under which optimal delegation takes the form of an optimal cap that is interior. We show that these conditions are easily checked when the extent of bias is small. In the second part of the paper, we apply the theory to the delegation of tariff-negotiation authority. With the legislative and executive branches playing the respective roles of principal and agent, we follow traditional arguments and assume that both branches are sensitive to pressure from import-competing interests but that the executive branch is less so. We also assume that the executive branch has private information about the probable effectiveness or the extent of the negotiated foreign tariff liberalization. We then apply the implications of delegation theory from the first part of the paper and provide conditions under which the optimal form of trade-policy authority for the negotiation of reciprocal tariff agreements entails a tariff floor with no money-burning used. We argue that this form of delegation is consistent with important features of the RTAA and subsequent legislation.

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1 Introduction

This paper explores the relationship between the theory of optimal delegation and the delegation of tariff-negotiation authority under U.S. legislation following the Reciprocal Trade Agreements Act (RTAA) of 1934.

The theory of optimal delegation addresses a setting in which a principal faces a biased but informed agent and contingent transfers between the principal and agent are not feasible. The principal then “delegates” authority to the agent, by selecting a set of permissible actions from which the agent may choose. The optimal delegation set (i.e., the optimal set of permissible actions) reflects a tradeoff for the principal between granting the agent flexibility so as to take advantage of the agent’s information versus restricting the set of permissible actions so as to limit the expression of the agent’s bias.

A central focus of the literature on delegation theory concerns the identification of settings under which “interval delegation” is optimal, whereby the optimal set of permissible actions is defined by an interval.\footnote{The delegation literature begins with Holmstrom’s (1977) seminal work. In addition to the papers discussed in this paragraph, other work on delegation theory includes Amador and Bagwell (2012, 2018), Amador et al (2018), Amador et al (2006), Ambrus and Egorov (2017), Frankel (2016) and Melumad and Shibano (1991).} Alonso and Matouschek (2008) consider a setting with quadratic welfare functions and give necessary and sufficient conditions under which interval delegation is optimal. Using Lagrangian methods, Amador and Bagwell (2013) consider a more general set of welfare functions and establish necessary and sufficient conditions under which interval delegation is optimal. They provide such conditions both for the traditional delegation problem with a one-dimensional delegation set and also for a modified delegation problem with a two-dimensional set, where in the modified problem an action may be permitted only when accompanied by an associated level of “money burning.” The former (latter) problem is referred to as the delegation problem without (with) money burning. The optimality of interval delegation for the delegation problem with money burning means that optimal delegation does not utilize a money-burning requirement even though such a requirement is feasible.

In this paper, we specify general welfare functions for the principal and agent into which a bias parameter enters in a separable fashion. This modeling framework allows us to vary the extent of bias in a simple way while still considering a general class of welfare functions, and we may think of the framework as applying to settings in which the principal and agent have related but distinct preferences. An important feature of the modeling framework is that it fits within the family considered by Amador and Bagwell (2013); thus, we are able to explore the implications of the general conditions developed in that paper for the modeling framework studied here.

We focus here on interval delegation that takes the form of a cap allocation, whereby
the agent is permitted to take any action below the cap and money burning is not required even when feasible. A cap allocation is interior when the cap binds only for states of nature above a threshold level, where the threshold level is interior within the set of possible states.

For the delegation problems with and without money burning, we apply the findings of Amador and Bagwell (2013) to the described class of welfare functions and report corresponding conditions under which an interior cap allocation is optimal. Under our assumptions, this means that optimal delegation takes a simple form: the agent’s flexible (i.e., preferred) action is selected for states below the optimal threshold value and the cap is selected for states above this value. Our results are particularly strong for settings in which the level of bias is small. If the agent is biased in favor of higher actions and the level of bias is sufficiently small, then we show that optimal delegation takes the form of an interior cap allocation provided only that the density over states is non-decreasing (e.g., a uniform density suffices).

Our finding here is related to Alonso and Matouschek’s (2008) Proposition 4, which concerns the optimality of interval allocations when preferences are sufficiently aligned. By comparison, we focus specifically on the optimality of cap allocations, consider a distinct family of welfare functions and allow for the possibility of money burning.2

We next turn to the application of our findings and consider the delegation of tariff-negotiation authority under U.S. legislation following the RTAA. Under the RTAA and subsequent legislation, the legislative branch (Congress) delegated the authority to negotiate reciprocal tariff agreements to the executive branch for periods of time (and subject to renewal) and with some restrictions. For our purposes, the key restrictions arose in two ways, both of which led to tariff floors (i.e., restrictions against tariffs below prescribed levels).

First, the U.S. legislative branch has delegated authority to the executive branch to negotiate reciprocal tariff agreements with the stipulation that the extent to which U.S. tariffs may be cut cannot exceed a given percentage. For example, in the original RTAA of 1934, tariffs on particular goods could be reduced by up to 50% from their 1934 level; in the 1955 RTAA extension, tariffs could be reduced by up to 15% (in three annual installments of 5%), although very high tariffs could be reduced by up to 50%; and in the 1958 renewal of the RTAA, tariffs could be reduced by up to 20%, with larger tariff cuts

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2 In our analysis, welfare functions may depend upon the agent’s action in a general way, and the principal and agent have similar but distinct welfare functions. For their main analysis, Alonso and Matouschek assume that welfare functions are quadratic in the agent’s action but impose less structure on the way in which the welfare functions of the principal and agent differ. (See Amador and Bagwell (2013) for further discussion on the former point.) The welfare functions that Alonso and Matouschek specify for their economic applications (in which the principal’s preferred action is linear in the state) are special cases of the welfare functions considered here. See footnote 22 for further discussion.
allowed for tariffs above 50%. The Trade Expansion Act of 1962 provided the executive branch with the authority to reduce tariffs across the board by up to 50%; the Trade Act of 1974 permitted import tariffs to be reduced by up to 60% and eliminated for tariffs under 5%; and the Omnibus Trade and Competitiveness Act of 1988 allowed the executive branch to reduce tariffs by up to 50%.

Second, some but not all renewals of the RTAA came with a “peril-point” provision. As Irwin (2017, p. 502) explains, under this provision, the president was required to “submit the list of goods that might be subject to tariff reductions to the Tariff Commission in advance of trade negotiations. The commission would then report on the maximum allowable reduction that could be made without inflicting serious harm on domestic producers in the industry.” The provision did not prevent a negotiated tariff reduction from going below the peril point, but in this case the president would be required to provide a report to Congress declaring and explaining the decision to do so. For example, peril-point provisions were included in the 1948 renewal of the RTAA, the 1951 renewal of the RTAA, and the 1958 renewal of the RTAA.

Thus, the RTAA and subsequent legislation created tariff floors through general tariff-reduction limits and, in some cases, peril-point provisions for goods under negotiation. As noted, the executive branch could go beyond the peril-point limit, but only if “the action and the reasons for it were made public” (Leddy and Norwood, 1963, p. 131). The constraint appears to have been effective in important cases; for example, Irwin (2017, p. 521) describes negotiations between the EEC and the US in the GATT Dillon Round (1961-62) and explains that the US was unable to “match the 20 percent offer [from the EEC] for fear of violating the peril point provision; although that provision constituted no real legislative constraint, there would be a political cost to going beyond it.”

To interpret the delegation of tariff-negotiation authority in terms of our delegation model, we regard the legislative branch as the “principal” and the executive branch as the “agent.” From this perspective, we may understand the RTAA and subsequent legislation as describing a “delegation contract” for the executive branch indicating the permitted set of tariffs. A possible way to interpret the peril-point provisions is then in terms of a two-dimensional delegation set, whereby the executive branch could select a tariff below the peril-point limit but only if a political cost is incurred. Such a cost would be similar

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3See Irwin (2017, pp. 430-1; 515; 520).
6There also exist statutes that describe circumstances under which the executive branch is empowered to raise tariff rates. By contrast, in this paper, we consider only delegated trade-policy authority as it relates to tariff reductions achieved through reciprocal tariff agreements.
7See also Leddy and Norwood (1963, p.145-6).
to the money-burning possibility in our delegation model, if the principal also finds the resulting administrative process costly.

We can also imagine that the legislative and executive branches have similar but distinct welfare functions. While both branches are sensitive to pressure from import-competing firms, the president represents a national electoral base and is thus more likely to regard trade policy through a broad lens that emphasizes consumer and exporter interests alongside import-competing interests. According to this traditional argument, as Irwin (2017, p. 432) explains, the president is thus “more likely to favor moderate tariffs than the Congress.” We may also expect that the executive branch has relevant private information, or at least non-verifiable information, that is perhaps received as a consequence of being directly involved in the negotiation of the trade agreement.

To go further in exploring the relationship between delegation theory and the delegation of tariff-negotiation authority, we must develop an applied model of trade and tariff negotiations. We begin by presenting a general modeling framework for the trade-policy application. We then illustrate the key features of the framework in the context of a partial-equilibrium model of trade with two countries, where the legislative branch in the “home” country delegates authority to the executive branch. Both branches maximize a weighted national welfare function and give additional weight to the interests of import-competing firms; however, the legislative branch attaches a larger welfare weight to those interests. Given these welfare functions, the executive branch favors lower tariffs more than does the legislative branch.

We assume further that, during the course of the negotiation, the executive branch acquires private information about the negotiation partner (i.e., the “foreign” country). While this assumption can be captured in many ways, we focus here on a setting in which the executive branch acquires information about the probable effectiveness of the partner’s negotiated tariff liberalization. For example, the executive branch may have superior information about the possible “loopholes” in the agreement that might allow the foreign country to limit home-country exports through quotas, the classification of products, the use of secondary protective instruments corresponding to non-tariff barriers, or implementation delay. We can thus think of the executive as having private information as to the

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8See, e.g., Irwin (2017, p. 442) for broadly supportive discussion. Irwin notes that within the executive branch the U.S. State Department “went to great lengths to avoid harming domestic producers by reducing tariffs more on imports that did not compete with domestic production.”

9For example, Irwin (2017, p. 489) provides a quote from Senator Eugene Millikin in 1949 expressing such concerns: “...foreign nation beneficiaries have circumvented their concessions by various devices, such as state trading, import quotas, bilateral agreements, preference systems, import licenses, and exchange restrictions.” Similarly, in his review of the bilateral agreements following the RTAA of 1934, Tasca (1967, p. 59) explains that the executive branch must assess the concessions that may be asked when non-tariff trade barriers “such as import and customs quotas, clearing and compensation arrangements, and exchange controls” are “particularly restrictive of American trade.” Likewise, referring to the early
probability that a meaningful gain from the negotiated foreign tariff liberalization will be realized.

We then show that the applied model can be captured using the welfare functions that we posit for the theory of optimal delegation. We can represent the applied model as a delegation model fitting our framework by regarding the “action” as the home-country welfare gain for its export good when the negotiated foreign tariff liberalization is effective. The agent has private information about the probability that this gain will be realized. The principal and agent welfare functions are closely related, but the agent is biased toward higher actions (or equivalently, toward greater reciprocal negotiated tariff liberalization). We then apply our propositions and establish conditions under which optimal delegation takes the form of an interior cap allocation for this action. Since the action is inversely related to the level of reciprocal negotiated tariff liberalization, we may equivalently interpret the propositions as establishing conditions under which optimal delegation takes the form of an interior tariff floor allocation. We note that, under these conditions, optimal delegation does not entail money burning, even when it is feasible. Thus, to the extent that the “political cost” of violating peril-point provisions is a wasteful cost, our model suggests that the optimal delegation contract would be designed in such a way that the agent would not have incentive to select a tariff below the relevant peril point.

The applied model that we put forth captures the negotiated foreign tariff liberalization in a reduced-form way; specifically, the executive branch of the home country selects its negotiated tariff level and then the negotiated foreign tariff level is induced in a mechanical fashion, according to an embedded reciprocity assumption. We thus sidestep any possibility of strategic delegation, whereby the delegation contract is designed to alter the behavior of the foreign country. While strategic delegation is an important and interesting topic, our focus here is on the delegation problem that arises within a country and across different branches of the domestic government. In this way, our applied analysis also differs from that in Amador and Bagwell (2013), who consider the delegation problem that arises between countries when they form a trade agreement that restricts their subsequent behavior within the agreement.

We also explore alternative interpretations of our applied model of trade and tariff negotiations. We show that the welfare functions in our partial-equilibrium model can

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GATT era, Curzon (1966, pp. 70-1, 85-6) argues that the effect during this era of European tariff cuts was hampered by the use of quantitative restrictions. See also Pomfret (1997, p. 82). Finally, we note that broad foreign product classifications may limit home-country exports by enhancing the extent to which non-discriminatory foreign tariff cuts provide benefits to competing exporters in third countries.

We also note that strategic delegation may be ineffective if the delegation contract is imperfectly observed by the foreign country or if the home country has private information about its commitment to the delegation contract. For more on strategic delegation and robustness issues, see Bagwell (1995, 2018), Fershtman and Judd (1987) and Katz (1991).
alternatively be interpreted as describing a setting in which the executive branch has private information about the extent of the negotiated foreign tariff liberalization. In other words, the executive branch may have private information as to the extent of reciprocity. For example, if there are multiple products, the executive branch may have private information about the fraction of products for which it is feasible for the foreign country to agree to the negotiated tariff cut. Similarly, if different import goods for the home country are exported by different foreign countries, then the executive branch may have private information about the fraction of foreign countries for which a negotiated tariff cut would be feasible (or effective).

In addition to research already mentioned, our work is related to a literature that examines the rationale for and implications of the delegation of tariff-negotiation authority. Focusing on the RTAA, Bailey et al (1997) assume that tariffs are set unilaterally when under Congressional control, so that the benefits of reciprocal (or "bundled") tariff reductions become feasible only when the President has authority to negotiate trade agreements. Assuming that the President prefers lower tariffs, they argue that Congress chooses to give the President a minimum tariff that then becomes the outcome of the agreement. Lohmann and O’Halloran (1994) argue that Congress adopted the RTAA as a means of escaping from inefficient tariff setting due to the “log-rolling” that occurs when tariffs are under Congressional control. They also examine constrained delegation taking the form of fast-track procedures rather than as a restricted set of permitted tariffs as in our analysis.

Our approach is complementary to these papers but highlights different considerations. We explore a model in which negotiations over tariffs are conducted by the executive branch, and we do not address the political process under which tariffs would be set were they controlled by the legislative branch. Our focus is on the permissible set of tariffs and possible accompanying money-burning administrative processes that the legislative branch chooses when the executive branch prefers lower tariffs and acquires private (or at least non-verifiable) information as to the probable effectiveness or extent of reciprocity by the foreign negotiation partner or partners. An implication of our analysis is that the negotiated tariff is sometimes above the tariff floor, a finding that is broadly consistent, for

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11 A related but distinct possibility is that the executive branch may acquire private information about the fraction of products for which the benefit to home-country exporters of a non-discriminatory foreign tariff cut is not significantly lessened due to the presence of competing third-country exporters. See Tasca (1967, pp. 165-6) for related discussion.

12 Fast-track procedures arose in the US through the Trade Act of 1974 as trade agreements (and particularly the 1973-79 Tokyo Round of GATT) addressed non-tariff barriers, leading to changes in domestic law requiring Congressional approval. See Irwin (2017, pp. 550-2). The central question in research on fast-track procedures concerns why Congress decided to express its approval (or disapproval) via fast-track procedures (i.e., a simple up-or-down majority vote with no amendments allowed). For different perspectives, see Celik et al (2015, 2018) and Conconi et al (2012).
example, with Irwin’s (2017, pp. 485-9) description of the 1947 Geneva Round of GATT and with Tasca’s (1967) analysis of US negotiated tariffs cuts following the RTAA.\textsuperscript{13} The possibility of private information held by negotiators is consistent with the lack of public information as to negotiated foreign tariff cuts for early GATT rounds and different measures of reciprocity used by delegations when reporting gains from negotiation to their respective governments.\textsuperscript{14}

O’Halloran’s (1994) model of delegated trade-policy authority is also broadly related. For a model with quadratic preferences, she assumes that the legislative and executive branches each have fixed ideal trade policies, where the executive branch prefers freer trade than does the legislative branch. The legislative branch delegates to the executive branch the power to change policy by a fixed amount $d \geq 0$, where a higher value for $d$ means greater discretion. After this decision is made, the state of the world is realized in that the “status quo” policy is determined. The set of permissible policies for the executive branch is thus defined by an interval of width $2d$ centered around the status quo policy.\textsuperscript{15} By contrast, we consider a more general family of preferences and distribution functions, allow that the ideal points depend on the state variable, and provide conditions under which an interval allocation (in the form of a tariff floor) is optimal among all incentive-compatible allocations, with and without the possibility of money burning.\textsuperscript{16}

The paper is organized as follows. Section 2 presents our delegation model and characterizations of optimal delegation. Section 3 presents an applied model of the delegation of tariff-negotiation authority for tariff negotiations. Section 4 concludes. Omitted proofs are in the Appendix.

\textsuperscript{13}As Irwin observes, while the executive branch had authority to cut tariffs by up to 50%, the average tariff reduction was 21%. Similarly, Tasca (1967, p. 185-6) reports that “by far the greatest number of duty reductions stipulate cuts of 20 percent or over,” where a “considerable number of these reductions are for the maximum 50 percent permitted by law.”

\textsuperscript{14}See Irwin (2017, p. 486) for further discussion regarding the lack of readily available information as to foreign tariff cuts in early GATT Rounds. (We note that bilateral negotiating material from early GATT Rounds of negotiation has recently been derestricted and is now available on the WTO website.) See Curzon and Curzon (1976, pp. 160-1) for discussion of different measures of reciprocity sometimes used by delegations when reporting to their respective governments.

\textsuperscript{15}O’Halloran also explores the empirical implications of her model, including the prediction that the degree of discretion granted to executive branch is greater when the preferences of the legislative and executive branches are more similar.

\textsuperscript{16}Since ideal points depend on the privately observed, or at least non-verifiable, state variable, the design of our model removes the possibility that the legislative branch could construct a delegation contract that simply specifies that the policy must equal its ideal point.
2 Delegation Theory

In this section, we develop and analyze a delegation model with a principal and agent, in which the two players have related but distinct welfare functions. We first develop the model and present the maintained assumptions. Next, we do some preliminary work and record some basic features of the model. Finally, we provide characterization results for the form of optimal delegation.

2.1 Model

Let \( \omega(\pi, \gamma, \phi) - t \) and \( \omega(\pi, \gamma, \phi_A) - t \) denote the respective welfare functions of the principal and agent, where

\[
\omega(\pi, \gamma, \phi) = b(\pi, \phi) + \gamma \pi
\]

and

\[
b(\pi, \phi) = \beta(\pi) + \phi \Lambda(\pi)
\]

and where \( \phi \in \{\phi_P, \phi_A\} = \Phi \) with \( \phi_A \neq \phi_P \). The value of \( \pi \) represents an action taken by the agent who is privately informed about the value of \( \gamma \). The value of \( t \) corresponds to a “money-burning” action that is a wasteful transfer, in that it symmetrically reduces the welfare of both the principal and the agent. Below, we consider both the case where money burning is allowed and when it is not.

We assume that \( \gamma \) has a twice continuously differentiable distribution \( F \) with bounded support \( \Gamma = [\gamma, \overline{\gamma}] \) where \( \underline{\gamma} < \gamma \) and with a strictly positive density \( f = F' \). Given the distribution \( F \), let \( E\gamma \in (\gamma, \overline{\gamma}) \) denote the expected value of \( \gamma \). The action \( \pi \) is selected from a set \( \Pi \), which is an interval on the real line having non-empty interior. For simplicity, we assume that \( \Pi \) is compact with \( \Pi = [0, \pi] \) where \( \pi > 0 \).\(^{17}\) We further maintain the following assumption:

**Assumption 1:** The following holds: (i) the functions \( \beta : \Pi \to \mathbb{R} \) and \( \Lambda : \Pi \to \mathbb{R} \) are twice continuously differentiable on \( \Pi \); (ii) for any \( \phi \in \Phi \), the function \( b : \Pi \times \Phi \to \mathbb{R} \) is strictly concave on \( \Pi \); and (iii) there exists a function \( \pi_f : \Gamma \times \Phi \to \Pi \) such that, for \( \phi = \phi_A \) and for all \( \gamma \in \Gamma, \pi_f(\gamma, \phi_A) \in (0, \pi) \), \( \pi_f(\gamma, \phi_A) \) is twice differentiable on \( \Gamma \), \( \frac{\partial \pi_f(\gamma, \phi_A)}{\partial \gamma} > 0 \) and \( \pi_f(\gamma, \phi_A) \) is the unique maximizer of \( \gamma \pi + b(\pi, \phi_A) \) over \( \pi \in \Pi \).

We note that \( \pi_f(\gamma, \phi_A) \) indicates the agent’s preferred or flexible action for any \( \gamma \).\(^{18}\)

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\(^{17}\) The model can be extended to allow that \( \pi = \infty \). See Amador and Bagwell (2013).

\(^{18}\) Similarly, \( \pi_f(\gamma, \phi_P) \) represents the action that would be preferred by the principal, were the principal to know \( \gamma \). As apparent from Assumption 1, we do not place additional structure on \( \pi_f(\gamma, \phi_P) \) and, in particular, allow that \( \pi_f(\gamma, \phi_P) \) may fail to be interior for some \( \gamma \in \Gamma \).
Given Assumption 1, the function $b : \Pi \times \Phi \to \mathbb{R}$ is bounded and twice continuously differentiable on $\Pi \times \Phi$ and, for any $\phi \in \Phi$, strictly concave on $\Pi$. We note that Assumption 1 does not impose separate concavity requirements on the functions $\beta : \Pi \to \mathbb{R}$ and $\Lambda : \Pi \to \mathbb{R}$. Assumption 1 similarly implies that the function $\omega : \Pi \times \Gamma \times \Phi \to \mathbb{R}$ is twice continuously differentiable on $\Pi \times \Gamma \times \Phi$, and that for any $\gamma \in \Gamma$ and $\phi \in \Phi$ the function $\omega$ is strictly concave on $\Pi$. With these observations in hand, we can confirm that Assumption 1 in Amador and Bagwell (2013) is satisfied when applied to the welfare functions as defined here in (1) and (2).

Following Amador and Bagwell (2013), we say that an allocation is a pair of functions $(\pi, t)$, with $\pi : \Gamma \to \mathbb{R}$ and $t : \Gamma \to \mathbb{R}$, that indicates the action and amount of money burned as a function of the private information, $\gamma$. We may understand an allocation as defining the set of permissible actions for the agent. The delegation problem then amounts to choosing the allocation $(\pi, t)$ that maximizes the principal’s expected welfare while satisfying an incentive-compatibility constraint for the agent and a feasibility constraint on the range of the money-burning function $t$.

We consider two problems. The delegation problem with money burning is defined as

$$\max_{\pi, t} \int_\gamma (\omega(\pi(\gamma), \gamma, \phi_p) - t(\gamma))dF(\gamma) \text{ subject to } (P)$$

$$\gamma \in \arg \max_{\gamma \in \Gamma} \{\omega(\pi(\tilde{\gamma}), \gamma, \phi_A) - t(\tilde{\gamma})\}, \ \forall \gamma \in \Gamma,$$

$$t(\gamma) \geq 0, \ \forall \gamma \in \Gamma,$$

where the first constraint is the agent’s incentive-compatibility constraint and the second constraint indicates that money can be burned but not generated. We are also interested in the optimal form of delegation when money burning is not allowed. The delegation problem without money burning is defined as Problem P with an additional constraint:

$$t(\gamma) = 0, \ \forall \gamma \in \Gamma. \ \ (3)$$

A defining feature of the delegation problem is that contingent transfers between the principal and agent are not allowed. This feature has important implications for the set of incentive-compatible allocations. For example, in the problem in which money burning is not allowed, an incentive-compatible allocation can be strictly increasing and continuous over an interval of values for $\gamma$ only if the allocation over that interval is defined by the agent’s flexible allocation, $\pi_f(\gamma, \phi_A)$. Incentive-compatible allocations may also then

19 The assumptions above strengthen those made by Amador and Bagwell (2013), in that we impose sufficient structure so that $\beta, \Lambda, \beta', \Lambda', \beta'', \Lambda'', f$ and $f'$ are bounded on their respective domains. These assumptions facilitate arguments that we make below concerning the satisfaction of certain properties for $|\phi_p - \phi_A|$ sufficiently small.
exhibit jump discontinuities from a lower step to a higher step, where a step is an interval of values for \( \gamma \) along which agent types are pooled (i.e., along which the allocation is independent of \( \gamma \)). For the critical \( \gamma_c \) at which such a jump occurs, the agent is indifferent between the allocations on the lower and higher steps, with the former (latter) being below (above) \( \pi_f(\gamma_c, \phi_A) \). The problem of finding an optimal allocation is thus non-trivial, due to the presence of discontinuous allocations in the feasible choice set.

To analyze the delegation problem, it is important to identify the direction of any bias in the agent’s welfare function. Using (1) and (2), we may easily confirm that

\[
\omega(\pi, \gamma, \phi_P) = \omega(\pi, \gamma, \phi_A) + (\phi_P - \phi_A)\Lambda(\pi).
\]

(4)

Differentiating and evaluating at the agent’s flexible choice, \( \pi_f(\gamma, \phi_A) \), we thus find that

\[
\omega\pi(\pi_f(\gamma, \phi_A), \gamma, \phi_P) = (\phi_P - \phi_A)\Lambda'(\pi_f(\gamma, \phi_A)).
\]

(5)

From (5), we see that, at a given \( \gamma \), the principal would benefit were the agent to slightly lower (raise) the action from the agent’s flexible level when \( \omega\pi(\pi_f(\gamma, \phi_A), \gamma, \phi_P) < 0 \) (\( > 0 \)). We thus say that the agent has an upward bias at \( \gamma \) when \( \omega\pi(\pi_f(\gamma, \phi_A), \gamma, \phi_P) < 0 \) and that the agent has a downward bias at \( \gamma \) when \( \omega\pi(\pi_f(\gamma, \phi_A), \gamma, \phi_P) > 0 \). Finally, we observe from (5) that whether the agent has upward or downward bias at \( \gamma \) is determined by the sign of \( (\phi_P - \phi_A)\Lambda'(\pi_f(\gamma, \phi_A)) \) at \( \gamma \).

2.2 Basic Features

We now record some basic features of the model. Our goal is to provide formal and intuitive foundations for the analysis of optimal delegation that follows in Section 2.3.

2.2.1 Optimal cap allocations

We are interested to determine conditions under which the delegation problem with or without money burning is solved by a cap allocation. To prepare for this determination, we now define a cap allocation and characterize its optimal form.

A cap allocation is defined by (i) a bound \( \gamma_H \in [\gamma, \overline{\gamma}] \) such that \( \pi(\gamma) = \pi_f(\gamma, \phi_A) \) for \( \gamma \in [\gamma, \gamma_H] \) and \( \pi(\gamma) = \pi_f(\gamma_H, \phi_A) \) for \( \gamma \in [\gamma_H, \overline{\gamma}] \), and (ii) \( t(\gamma) = 0, \forall \gamma \in \Gamma \).

It is straightforward to see that cap allocations are incentive compatible and thus satisfy the constraints of Problem P. Of course, a cap allocation cannot be optimal in

\cite{Melumad and Shibano 1991} characterize incentive-compatible allocations when preferences are quadratic and money burning is not allowed. When money burning is allowed, we note that incentive-compatible allocations include action functions \( \pi(\gamma) \) that are strictly increasing and continuous over an interval of values for \( \gamma \) while also differing from the agent’s flexible action function \( \pi_f(\gamma, \phi_A) \) over that interval.

\footnote{Melumad and Shibano (1991) characterize incentive-compatible allocations when preferences are quadratic and money burning is not allowed. When money burning is allowed, we note that incentive-compatible allocations include action functions \( \pi(\gamma) \) that are strictly increasing and continuous over an interval of values for \( \gamma \) while also differing from the agent’s flexible action function \( \pi_f(\gamma, \phi_A) \) over that interval.}
the full set of incentive-compatible allocations unless it is optimal in the restricted set of cap allocations. We thus begin by defining and characterizing cap allocations that are optimal within the restricted class of cap allocations.

We say that a cap allocation defined by the bound $\gamma^*_H \in [\gamma, \bar{\gamma}]$ is an optimal cap allocation if $\gamma^*_H$ achieves the maximum value for the objective in Problem P when the objective is maximized over the restricted set of cap allocations. An optimal cap allocation is interior if $\gamma^*_H \in (\gamma, \bar{\gamma})$.

Notice that the problem of characterizing an optimal cap allocation amounts to an optimization problem over $\gamma_H \in \Gamma$. Thus, we can use standard calculus techniques for characterizing an optimal cap allocation, $\gamma^*_H$.

We now provide a characterization of an optimal cap allocation that is interior.

**Proposition 1** Assume

$$\omega_{\pi}(\pi f(\gamma, \phi_A), \gamma, \phi_P) = (\phi_P - \phi_A)\Lambda'(\pi f(\gamma, \phi_A)) < 0$$

so that the agent has an upward bias at $\gamma = \bar{\gamma}$. Assume further that

$$\omega_{\pi}(\pi f(\gamma, \phi_A), \gamma, \phi_P) + E\gamma - \gamma = (\phi_P - \phi_A)\Lambda'(\pi f(\gamma, \phi_A)) + E\gamma - \gamma > 0,$$

which is assured if $|\phi_P - \phi_A| > 0$ is sufficiently small. Then there exists an optimal cap allocation that is interior: $\gamma^*_H \in (\gamma, \bar{\gamma})$.

The proof of Proposition 1 is in the Appendix.

**2.2.2 Intuition**

Proposition 1 provides conditions under which there exists an optimal cap allocation that is interior. We seek to go further and determine conditions under which an optimal cap allocation that is interior is optimal over the full set of incentive-compatible allocations, both when money burning is allowed and when it is not. To this end, our approach is to refer to the general sufficient conditions for optimality as found in Amador and Bagwell (2013) and to confront those conditions with the welfare functions defined in (1) and (2). Before proceeding with this formal approach, we pause here to develop an intuitive foundation for understanding some of the key considerations at play.

Figure 1 illustrates the optimal cap allocation with an interior value for $\gamma^*_H$. In Figure 2, we depict an alternative allocation, which is identical to the optimal cap allocation except that a “hole” is drilled in the flexible region: for $\gamma < \gamma_1 < \gamma_2 < \gamma^*_H$, actions in the range from $\pi f(\gamma_1, \phi_A)$ to $\pi f(\gamma_2, \phi_A)$ are removed. The alternative allocation is incentive compatible provided that the depicted critical state, $\gamma_c \in (\gamma_1, \gamma_2)$, is defined as the state at
which the agent is indifferent between $\pi_f(\gamma_1, \phi_A)$ and $\pi_f(\gamma_2, \phi_A)$.

For current purposes, let us suppose further that the agent has an upward bias at all $\gamma \in \Gamma$. For any $\gamma \in \Gamma$, it then follows that the principal would prefer that the agent select an action at least slightly below the agent’s flexible level, $\pi_f(\gamma, \phi_A)$.

Of course, if the optimal cap allocation is optimal among all incentive-compatible allocations, then it must in particular generate higher expected welfare for the principal than does the alternative allocation. Given the assumption of upward bias, we now develop two considerations that bear on this comparison.

---

21 The principal could implement the alternative allocation by selecting a permissible set that allows for any action at or below $\pi_f(\gamma_H^*, \phi_A)$ except for actions in $(\pi_f(\gamma_1, \phi_A), \pi_f(\gamma_2, \phi_A))$. 

12
The first consideration relates to the slope of the density, $f$. Refer to Figure 2. For $\gamma \in (\gamma_1, \gamma_c)$, the alternative allocation results in a lower action, which generates a gain for the principal so long as the “hole” is not too large. For $\gamma \in (\gamma_c, \gamma_2)$, however, the alternative allocation results in a higher action, which generates a loss for the principal. Intuitively, we may thus expect that a nondecreasing density would make it more likely that the optimal cap allocation generates higher expected principal welfare than the alternative allocation.

The second consideration relates to the concavity of the principal’s welfare function relative to that of the agent. Referring again to Figure 2, we observe that the alternative allocation adds variation to the distribution of actions relative to the actions that lie along the agent’s flexible function, $\pi_f(\gamma, \phi_A)$. Since the slope of the agent’s flexible function is given by $-1/b_{\pi\pi}(\pi, \phi_A)$, we may expect that the optimal cap allocation is more likely to be preferred by the principal to the alternative allocation if $\omega_{\pi\pi}(\pi, \gamma, \phi_P)/b_{\pi\pi}(\pi, \phi_A)$ is not too small. Intuitively, such a “relative concavity” condition ensures that the principal does not value too much the increase in spread that the alternative allocation implies.

While the comparison above considers only one alternative allocation, it provides some background intuition for the sufficient conditions developed by Amador and Bagwell (2013). As described below, when the agent is upward biased at all $\gamma \in \Gamma$ and the “bias function” is independent of the state, their sufficient conditions include a nondecreasing density and a lower-bound restriction on the relative concavity of the principal’s welfare function. Our discussion here thus provides some motivation for the characterizations that follow in Sections 2.2.3 and 2.2.4 and also provides an intuitive foundation for the results on optimal delegation found in Section 2.3. As we will see in Section 2.3, a contribution of the current paper is to show further that Amador and Bagwell’s (2003) sufficient conditions are particularly easy to check for the welfare functions defined in (1) and (2) when the level of bias as measured by $|\phi_P - \phi_A|$ is sufficiently small.

2.2.3 Relative Concavity: $\kappa$

To state their sufficient conditions, Amador and Bagwell (2013) introduce a constant $\kappa$ that captures the relative concavity of the principal’s and agent’s welfare functions with respect to the action variable, $\pi$. As suggested above and as they explain, this value plays an important role in determining the optimality of interval allocations, such as cap allocations.

Following Amador and Bagwell (2013), we define

$$\kappa = \min_{(\gamma, \pi) \in \Gamma \times \Pi} \left\{ \frac{\omega_{\pi\pi}(\pi, \gamma, \phi_P)}{b_{\pi\pi}(\pi, \phi_A)} \right\}$$

(8)
for the delegation problem without money burning, and we define

$$\kappa = \min \left\{ \min_{(\gamma, \pi) \in \Gamma \times \Pi} \left\{ \frac{\omega_{\pi\pi}(\pi, \gamma, \phi_P)}{b_{\pi\pi}(\pi, \phi_A)} \right\}, 1 \right\} \quad (9)$$

for the problem with money burning.

We now consider the properties of the expressions for \( \kappa \) in (8) and (9) given the welfare functions defined in (1) and (2) and Assumption 1. An immediate observation is that for both problems \( \kappa > 0 \), since \( \omega_{\pi\pi}(\pi, \gamma, \phi_P) < 0 \) and \( b_{\pi\pi}(\pi, \phi_A) < 0 \) under Assumption 1.

Next, we have that

$$\omega_{\pi\pi}(\pi, \gamma, \phi_P) b_{\pi\pi}(\pi, \phi_A) = \omega_{\pi\pi}(\pi, \gamma, \phi_P) \omega_{\pi\pi}(\pi, \gamma, \phi_A) = 1 + (\phi_P - \phi_A) \Lambda''(\pi), \quad (10)$$

where the second equality uses (4). Since \( \omega_{\pi\pi}(\pi, \gamma, \phi_A) < 0 \) under Assumption 1, we see from (8)-(10) that \( \kappa < 1 \) (\( \kappa = 1 \)) in both problems if \((\phi_P - \phi_A) \Lambda''(\pi) > 0 \) ((\(\phi_P - \phi_A) \Lambda''(\pi) = 0 \)) for all \( \pi \in \Pi \). For a given value of \( \phi_A \), we notice further that \( \kappa \to 1 \) in both problems as \( \phi_P \to \phi_A \).

For convenient reference, we summarize our findings with the following proposition:

**Proposition 2** For the delegation problems with and without money burning, we have that (i) \( \kappa > 0 \), (ii) \( \kappa < 1 \) (\( \kappa = 1 \)) if \((\phi_P - \phi_A) \Lambda''(\pi) > 0 \) ((\(\phi_P - \phi_A) \Lambda''(\pi) = 0 \)) for all \( \pi \in \Pi \), and (iii) for a given value of \( \phi_A \), \( \kappa \to 1 \) as \( \phi_P \to \phi_A \).

For both problems, an immediate implication of Proposition 2 is that, for given \( \phi_A \), \( \kappa > 1/2 \) if \(|\phi_P - \phi_A|\) is sufficiently small. This implication will be useful below, when we apply results from Amador and Bagwell (2013).

### 2.2.4 The bias function, \( v \)

For certain results, Amador and Bagwell (2013) also find it useful to define a function \( v \) that equals the difference between the welfare function of the principal and that of the agent, \( \omega(\pi, \gamma, \phi_P) - \omega(\pi, \gamma, \phi_A) \). Using (4), we can easily represent this “bias function” for the welfare functions under consideration here. Specifically, we have that

$$v(\pi; \phi_P, \phi_A) = (\phi_P - \phi_A) \Lambda(\pi). \quad (11)$$

For the welfare functions considered here, we thus have that the function \( v \) is independent of the state variable, \( \gamma \).

The following proposition records some properties of the function \( v \) that will be useful when we apply the sufficient conditions of Amador and Bagwell (2013):
Proposition 3  (i) The principal’s and agent’s welfare functions satisfy \( \omega(\pi, \gamma, \phi_P) = \omega(\pi, \gamma, \phi_A) + v(\pi; \phi_P, \phi_A) \), where \( v(\pi; \phi_P, \phi_A) \) is defined by (11) and is independent of \( \gamma \); (ii) If there exists an optimal cap allocation that is interior, \( \gamma^*_H \in (\gamma, \bar{\gamma}) \), then

\[
v_\pi(\pi_f(\gamma_H, \phi_A); \phi_P, \phi_A) + E[\gamma | \gamma \geq \gamma_H] - \gamma_H = 0; \quad (12)
\]

and (iii) \( v_\pi(\pi_f(\gamma, \phi_A); \phi_P, \phi_A) \leq 0 \) for all \( \gamma \in [\gamma, \gamma_H] \) holds if and only if \( (\phi_P - \phi_A)\Lambda'(\pi_f(\gamma, \phi_A)) \leq 0 \) for all \( \gamma \in [\gamma, \gamma_H] \) holds.

The proof of Proposition 3 is in the Appendix.

2.3 Optimal Delegation

We are now prepared to apply results from Amador and Bagwell (2013) and determine conditions under which the delegation problem with and without money burning is solved by an optimal cap allocation, when the welfare functions take the form given by (1) and (2). Our approach is to use Proposition 3 of Amador and Bagwell (2013), which applies when, as here, the bias function \( v \) is independent of the state variable, \( \gamma \). A contribution of our work here is to utilize the form of the welfare functions as specified by (1) and (2) to identify general optimality conditions that emerge when the bias as defined by \( |\phi_P - \phi_A| \) is sufficiently small.

Proposition 3 in Amador and Bagwell (2013) establishes conditions under which an optimal cap allocation solves the delegation problem with and without money burning. For reference, we re-state that proposition here (with some slight modifications in notation to match with that in the current paper):

Proposition 4 (Amador and Bagwell, 2013). Assume that (i) \( \omega(\pi, \gamma, \phi_P) = b(\pi, \phi_A) + \gamma_\pi + \bar{\omega}(\pi) \) for some function \( \bar{\omega} : \Pi \to \mathbb{R} \); (ii) \( \kappa \geq 1/2 \) with \( \kappa \) defined by (8) (or alternatively by (9)); and (iii) there exists \( \gamma_H \in (\gamma, \bar{\gamma}) \) such that

\[
\bar{\omega}'(\pi_f(\gamma_H)) + E[\gamma | \gamma \geq \gamma_H] - \gamma_H = 0, \quad (13)
\]

and \( \bar{\omega}'(\pi_f(\gamma)) \leq 0 \) for all \( \gamma \in [\gamma, \gamma_H] \). Then, for \( f \) nondecreasing, the cap allocation defined by \( \gamma_H \) solves Problem P with the additional constraint (3) (or alternatively, solves Problem P).

We note that Proposition 4 is in line with the intuitive considerations highlighted in Section 2.2.2, in that it features a nondecreasing density and a lower bound \( (\kappa \geq 1/2) \) on the relative concavity of the principal’s welfare function.

As shown in Proposition 3, the model considered here satisfies \( \omega(\pi, \gamma, \phi_P) = \omega(\pi, \gamma, \phi_A) + v(\pi; \phi_P, \phi_A) \), where \( v(\pi; \phi_P, \phi_A) \) is defined by (11) and is independent of \( \gamma \). The model
thus satisfies condition (i) in Proposition 4 when we set $\bar{v}(\pi) = v(\pi; \phi_P, \phi_A)$ and recall that $\omega(\pi, \gamma, \phi_A) = b(\pi, \phi_A) + \gamma \pi$ by (1). Next, we note that, if there exists an optimal cap allocation that is interior, $\gamma_H^* \in (\gamma, \overline{\gamma})$, then with $\bar{v}(\pi) = v(\pi; \phi_P, \phi_A)$ and $\gamma_H = \gamma_H^*$ we have from (12) in Proposition 3 that (13) as defined in condition (iii) of Proposition 4 must hold.

Using these observations, we may use Proposition 4 to state the following proposition:

**Proposition 5** Assume that (i) there exists an optimal cap allocation that is interior, $\gamma_H^* \in (\gamma, \overline{\gamma})$; (ii) $\kappa \geq 1/2$ with $\kappa$ defined by (8) (or alternatively by (9)); and (iii) $v_\pi(\pi_f(\gamma, \phi_A); \phi_P, \phi_A) \leq 0$ for all $\gamma \in [\gamma, \gamma_H^*]$. Then, for $f'(\gamma) \geq 0$, $\forall \gamma \in \Gamma$, the cap allocation defined by $\gamma_H^*$ solves the delegation problem without money burning (or alternatively, solves the delegation problem with money burning).

For a given specification of the model, the three conditions stated in Proposition 5 may be directly checked using the propositions above. Proposition 1 gives conditions (6) and (7), where the latter is sure to hold for given $\phi_A$ if $|\phi_P - \phi_A|$ is sufficiently small, which suffice for the existence of an optimal cap allocation that is interior. Likewise, Proposition 2 establishes that, for given $\phi_A$, $\kappa > 1/2$ holds if $|\phi_P - \phi_A|$ is sufficiently small. Finally, Proposition 3 shows that $v_\pi(\pi_f(\gamma, \phi_A); \phi_P, \phi_A) \leq 0$ for all $\gamma \in [\gamma, \gamma_H^*]$ holds if and only if $(\phi_P - \phi_A)\lambda'(\pi_f(\gamma, \phi_A)) \leq 0$ for all $\gamma \in [\gamma, \gamma_H^*]$ holds.

Using this approach, we show that when the level of bias as measured by $|\phi_P - \phi_A|$ is sufficiently small, we can establish very simple and easy-to-check sufficient conditions under which the delegation problems with and without money burning are solved by an optimal cap allocation that is interior.

**Proposition 6** Assume that (6) holds. For given $\phi_A$, assume further that $|\phi_P - \phi_A|$ is sufficiently small so that (7) and $\kappa > 1/2$ both hold. Assume also that $(\phi_P - \phi_A)\lambda'(\pi_f(\gamma, \phi_A)) \leq 0$ for all $\gamma \in [\gamma, \gamma_H^*]$. Then, for $f'(\gamma) \geq 0$, $\forall \gamma \in \Gamma$, the cap allocation defined by $\gamma_H^*$ solves the delegation problem without money burning (or alternatively, solves the delegation problem with money burning).

Using this proposition, we know that if the level of bias as measured by $|\phi_P - \phi_A|$ is sufficiently small and the density is nondecreasing, then the solution for the delegation problems with and without money burning is given by an optimal cap allocation that is interior, $\gamma_H^* \in (\gamma, \overline{\gamma})$, provided that (i) $(\phi_P - \phi_A)\lambda'(\pi_f(\gamma, \phi_A)) < 0$, and (ii) $(\phi_P - \phi_A)\lambda'(\pi_f(\gamma, \phi_A)) \leq 0$ for all $\gamma \in [\gamma, \gamma_H^*]$. Importantly, we note that conditions (i) and (ii) are assured if the agent has an upward bias at $\overline{\gamma}$ and does not have a downward bias at any $\gamma \in \Gamma$. For example, and as in the illustration in Section 2.2.2, the agent could have an upward bias for all $\gamma \in \Gamma$.

We thus have the following corollary:
Corollary 1 If the level of bias as measured by $|\phi_P - \phi_A|$ is sufficiently small, the agent has an upward bias at $\bar{\gamma}$ and does not have a downward bias at any $\gamma \in \Gamma$, and the density $f$ satisfies $f'(\gamma) \geq 0, \forall \gamma \in \Gamma$, then the delegation problems with and without money burning are both solved by an optimal cap allocation that is interior, $\gamma^*_H \in (\underline{\gamma}, \bar{\gamma})$.

Thus, for the welfare functions as specified by (1) and (2) and satisfying Assumption 1, if the level of bias is sufficiently small and the agent has upward bias across the full support of the state variable, then the delegation problems with and without money burning are both solved by an optimal cap allocation that is interior, provided only that the density is nondecreasing throughout the support. This result imposes structure on the sign and magnitude of bias but otherwise adds no further restrictions on the welfare functions; however, it does impose some structure on the monotonicity of the density, although this structure is satisfied by some popular examples (e.g., uniform distribution). For particular examples, it is also possible to relax the assumption of a monotone density.

3 Application to Tariff Negotiations

We now develop an application to tariff negotiations that utilizes the results from the previous section. We first present a general modeling framework for the application. Next, we illustrate key features of the framework in the context of a partial-equilibrium model of trade with general functional forms. We then specify a linear-quadratic version of the partial-equilibrium model and give precise conditions on exogenous parameters and model primitives under which our assumptions and results hold. Finally, we discuss alternative interpretations of our findings for the partial-equilibrium model.

3.1 General Framework

We imagine a “home” country in which the legislative branch delegates authority to the executive branch for the negotiation of a trade agreement with a “foreign” country. The

22 Alonso and Matouschek (2008) develop a related finding in their Proposition 4. Our model differs in two respects. First, we allow for the possibility of money burning. Second, the welfare functions we consider are more general in that they may depend on the agent’s action in general (non-quadratic) ways but are more restrictive in that we impose additional structure on the way in which the welfare functions of the principal and agent differ. The welfare functions that Alonso and Matouschek specify for their economic applications impose that the principal’s preferred action is linear in the state and are special cases of the welfare functions considered here for the setting without money burning. Specifically, as Amador and Bagwell (2013) argue, the welfare functions considered by Alonso and Matouschek can be represented as $\omega(\pi, \gamma, \phi_P) = -(\pi - \pi_P(\gamma))^2/2$ and $\omega(\pi, \gamma, \phi_A) = -\pi^2/2 + \gamma \pi$, where $\pi_P(\gamma)$ is the principal’s preferred action given the state $\gamma$. If $\pi_P(\gamma) = \gamma + \alpha$ for some constant $\alpha$ (with $\alpha = 0$ allowed), then these welfare functions fit into our framework with $\Lambda(\pi) = [\alpha \pi - (\gamma + \alpha)^2/2]^{1/2}/(\phi_P - \phi_A)$ and $\beta(\pi) = -\pi^2/2 - \phi_A \Lambda(\pi)$.

23 We discuss this point further at the end of Section 3.3.
legislative branch has different preferences than does the executive branch and delegates authority subject to some endogenous constraints; formally, the legislative branch determines a set of permissible tariffs for the home country to which the executive branch may agree in the reciprocal tariff negotiation. We allow as well for the possibility that certain tariffs may be permissible only when coupled with a wasteful money burning activity that lowers the welfare of the legislative and executive branches. To facilitate the link to the literature on delegation theory, we refer to the legislative branch as the “principal” and the executive branch as the “agent.”

To develop the tariff model, we begin by specifying the welfare functions of the principal and agent as \( \tilde{\omega}(\tau, \gamma, \phi_P) - t \) and \( \tilde{\omega}(\tau, \gamma, \phi_A) - t \), respectively, where

\[
\tilde{\omega}(\tau, \gamma, \phi) = \tilde{b}(\tau, \phi) + \gamma \tilde{\pi}(\tau),
\]

and

\[
\tilde{b}(\tau, \phi) = \tilde{\beta}(\tau) + \phi \tilde{\Lambda}(\tau)
\]

and where \( \phi \in \{\phi_A, \phi_P\} = \Phi \) with \( \phi_A \neq \phi_P \), \( \gamma \in [\underline{\gamma}, \overline{\gamma}] = \Gamma \) with \( \underline{\gamma} < \overline{\gamma} \), and \( \tau \in \Upsilon = [0, \tau] \) with \( \tau > 0 \). We interpret \( \phi \) as a preference parameter, \( \tau \) as the status quo tariff and \( \tau \) as the negotiated home import tariff. As before, \( t \) is a money burning variable where money burning may or may not be feasible. It is understood that \( t \equiv 0 \) when money burning is infeasible. The function \( \tilde{\pi}(\tau) \) can be interpreted in various ways; for example, it could indicate a negotiated benefit enjoyed by the home country regardless of preferences, where the size of the benefit is related to the negotiated home tariff. The state variable \( \gamma \) could then represent the probability that this benefit is realized or the scale of the benefit. We develop an interpretation of this kind below for a partial-equilibrium model.

To translate this modeling framework into the delegation model as defined and analyzed in the previous section, we must write welfare as a function of different arguments. To this end, we assume that

\[
\tilde{\pi}'(\tau) < 0 \quad \text{for all } \tau \in \Upsilon = [0, \tau]
\]

with

\[
\tilde{\pi}(0) = \pi > 0 = \tilde{\pi}(\tau).
\]

Given (16), we may invert \( \pi = \tilde{\pi}(\tau) \) to obtain the relation, \( \tau = \tilde{\tau}(\pi) \). We may thus re-state our assumptions (16) and (17) as

\[
\tilde{\pi}'(\pi) < 0 \quad \text{for all } \pi \in \Pi = [0, \overline{\pi}].
\]

More generally, we can proceed in this fashion provided that \( \tilde{\pi}(\tau) \) is strictly monotone and thus invertible. We assume in (16) that \( \tilde{\pi}(\tau) \) is strictly decreasing, since this ensures that the general framework includes the partial-equilibrium model developed below.
We can now recover the functions defined in the delegation model as follows:

\[ \omega(\pi, \gamma, \phi) = \beta(\pi) + \phi \Lambda(\pi) + \gamma \pi, \]

where

\[ \beta(\pi) = \hat{\beta}(\hat{\tau}(\pi)) \]
\[ \Lambda(\pi) = \hat{\Lambda}(\hat{\tau}(\pi)), \]

and where by construction \( \pi = \hat{\pi}(\hat{\tau}(\pi)) \) and \( \pi \in [0, \pi] = \Pi \) with \( \pi = \hat{\pi}(0) \) and \( 0 = \hat{\tau}(\pi) \).

As before, we have that

\[ b(\pi, \phi) = \beta(\pi) + \phi \Lambda(\pi) \]

so that

\[ \omega(\pi, \gamma, \phi) = b(\pi, \phi) + \gamma \pi. \]

We can now say that an allocation, \( \pi(\gamma) \), in the delegation model induces a corresponding tariff allocation, \( \tau(\gamma) = \hat{\tau}(\pi(\gamma)) \), in the tariff model. In particular, we recall that a cap allocation in the delegation model is defined by (i) a bound \( \gamma_H \in [\gamma, \bar{\gamma}] \) such that \( \pi(\gamma) = \pi_f(\gamma, \phi_A) \) for \( \gamma \in [\gamma, \gamma_H] \) and \( \pi(\gamma) = \pi_f(\gamma_H, \phi_A) \) for \( \gamma \in [\gamma_H, \bar{\gamma}] \), and (ii) \( t(\gamma) = 0, \forall \gamma \in \Gamma \). Letting the agent’s flexible tariff allocation be denoted as \( \tau_f(\gamma, \phi_A) \equiv \hat{\tau}(\pi_f(\gamma, \phi_A)) \) and recalling that \( \hat{\tau}'(\pi) < 0 \) for all \( \pi \in \Pi \), we see that a cap allocation \( \pi(\gamma) \) induces a tariff floor allocation in the tariff model, where the tariff floor allocation is defined by (i) a bound \( \gamma_H \in [\gamma, \bar{\gamma}] \) such that \( \tau(\gamma) = \tau_f(\gamma, \phi_A) \) for \( \gamma \in [\gamma, \gamma_H] \) and \( \tau(\gamma) = \tau_f(\gamma_H, \phi_A) \) for \( \gamma \in [\gamma_H, \bar{\gamma}] \), and (ii) \( t(\gamma) = 0, \forall \gamma \in \Gamma \).

In this way, a proposed tariff model of the form in (14) and (15) that satisfies (16) and (17) can be translated into the delegation framework considered in the previous section. We can then examine whether the resulting delegation model satisfies Assumption 1 and the other assumptions imposed in our propositions. If the propositions indicate that a cap allocation is optimal, then we may translate this finding back to the tariff model and conclude that the corresponding optimal tariff allocation takes the form of a tariff floor.

To facilitate the application of our propositions, we may represent the bias in the agent’s preferences in terms of the underlying tariff model. Recall that the bias function is given as \( v(\pi; \phi_P, \phi_A) = (\phi_P - \phi_A) \Lambda(\pi) \). Observe now that, for \( \pi \in \Pi \),

\[ v_\pi(\pi; \phi_P, \phi_A) = (\phi_P - \phi_A) \cdot \Lambda'(\pi) = (\phi_P - \phi_A) \cdot \hat{\Lambda}'(\hat{\tau}(\pi)) \cdot \hat{\tau}'(\pi) \]

where \( \hat{\tau}'(\pi) < 0 \) for \( \pi \in \Pi \) by (18).

Similarly, to evaluate the direction of the agent’s bias in terms of the underlying tariff
Thus, since \( \hat{\tau}'(\pi) < 0 \) for \( \pi \in \Pi \) by (18), we have that
\[
\text{sign}\{\omega_{\pi}(\pi f(\gamma, \phi_A), \gamma, \phi_P)\} = \text{sign}\{(\phi_P - \phi_A) \cdot \hat{N}'(\pi f(\gamma, \phi_A))\}
\]
(19)

for \( \pi f(\gamma, \phi_A) \in \Pi \). Thus, for example, the tariff model under (18) leads to a delegation model in which the agent has upward bias for all \( \gamma \in \Gamma \) if \( \phi_P > \phi_A \) and, for all \( \gamma \in \Gamma \), \( \hat{N}'(\pi f(\gamma, \phi_A)) > 0 \) and \( \pi f(\gamma, \phi_A) \in \Pi \).

### 3.2 Partial-Equilibrium Model

We now develop a simple partial-equilibrium model of trade and tariff negotiations. We show that the model provides a foundation for the assumption in the delegation model that the agent has upward bias for all \( \gamma \in \Gamma \).

We consider a two-country model of trade, in which the home country imports good \( x \), the foreign country imports good \( y \) and a numeraire good \( n \) is freely traded.\(^{25}\) For simplicity, we focus on a symmetric model. In each country, consumers have an identical quasi-linear utility function, given by
\[
u(x) = u(c_x) + u(c_y) + c_n,
\]
where \( c_i \) denotes the amount consumed of good \( i \) for \( i \in \{x, y, n\} \). We assume that \( u \) is strictly increasing, strictly concave and thrice continuously differentiable. The model features perfect competition. The supply functions for goods \( x \) and \( y \) in the home country are respectively given as
\[Q_x(p_x)\] and \( Q_y(p_y)\), where \( p_x \) and \( p_y \) are the associated prices of goods \( x \) and \( y \) in the home country. Letting \( p_x^* \) and \( p_y^* \) denote the corresponding prices in the foreign country, we assume that the foreign supply functions, \( Q_x^*(p_x^*)\) and \( Q_y^*(p_y^*)\), take a mirror image form so that \( Q_x(p) = Q_y^*(p_y) \) and \( Q_y(p) = Q_x^*(p) \). For prices that elicit strictly positive supply, we assume that these supply functions are strictly increasing and twice continuously differentiable. Finally, for any price \( p \) such that world supply is positive, we assume that \( Q_x(p) < Q_x^*(p) \); thus, the home country imports (exports) good \( x \) (\( y \)) under global free trade.

As is standard, we assume that the numeraire good is produced and consumed in strictly positive quantities in each country, where production occurs under conditions of

\(^{25}\)See, for example, Amador and Bagwell (2013) for the development of a similar model of trade.
perfect competition with constant returns to scale. In each country, labor is the only factor, and the supply of labor is perfectly inelastic. We may thus set the wage and price of the numeraire equal to one. The numeraire good is freely traded, with the level of trade determined so as to achieve trade balance.

We assume that each country selects a specific (per unit) import tariff. The home country thus applies a tariff, $\tau \in [0, \overline{\tau}] = \Upsilon$ where $\overline{\tau} > 0$, and the foreign country similarly has a tariff, $\tau^\ast$, where $\tau^\ast \in \Upsilon$. As we detail in the Appendix, the market-clearing local price for good $x$ in each of the two countries is determined as a function of $\tau$ by the requirements that the home import volume equals the foreign export volume and that $p_x = p_x^* + \tau$. In similar fashion, the market-clearing local price for good $y$ in each of the two countries is determined as a function of $\tau^\ast$. We assume that trade volumes are strictly positive and in the posited direction for all $(\tau, \tau^\ast) \in \Upsilon^2$.

For given tariffs and the resulting market-clearing prices, and as we detail in the Appendix, we can calculate the home-country consumer surplus and producer surplus for each of goods $x$ and $y$, and we can likewise determine the home-country tariff revenue. We denote the consumer surplus functions as $\hat{CS}_x(\tau)$ and $\hat{CS}_y(\tau^\ast)$, the producer surplus functions as $\hat{PS}_x(\tau)$ and $\hat{PS}_y(\tau^\ast)$, and the tariff revenue function as $\hat{TR}(\tau)$.

We consider a setting in which the legislative branch of the home country acts as the principal and the executive branch acts as the agent, where the producer surplus in the import-competing sector, $\hat{PS}_x(\tau)$, is given a special welfare weight represented by $\phi_P \geq 1$ for the legislative branch and by $\phi_A \geq 1$ for the executive branch. We capture the negotiation process in a reduced-form way with an embedded reciprocity assumption whereby a lower home negotiated tariff $\tau$ mechanically elicits a lower foreign negotiated tariff $\tau^\ast$. We assume, however, that the agent has private information about the probable extent to which the effective level of foreign protection will fall. For example, based on the negotiation history, the agent may have superior information about the possible "loopholes" in the agreement that might allow the foreign country to limit home-country exports through quotas, the classification of products, the use of secondary protective instruments corresponding to non-tariff barriers, or implementation delay.

Formally, we operationalize the tariff model as follows. Since the countries are symmetric, we assume that they start with the same tariff, $\overline{\tau}$, and that the home country then proposes a symmetric tariff level $\tau \in [0, \overline{\tau}] = \Upsilon$ for both countries, which the foreign country accepts. At the time of the negotiation, the agent has private information about the probability $\gamma \in \Gamma = [0, 1]$ that the negotiated foreign liberalization will be effective. With probability $\gamma$, the negotiated foreign liberalization is effective, and the prices and resulting welfare functions are determined based on the tariff pair $(\tau, \tau^\ast) = (\tau, \tau)$. The negotiated foreign liberalization is ineffective with complementary probability $1 - \gamma$, however, and in that case the effective level of foreign protection returns to $\overline{\tau}$, so that the
final tariff pair to which the market responds is \((\tau, \tau^*) = (\tau, \tau)\).  

We also set \(\tau\) so that it is the unilaterally optimal and, in this setting, Nash tariff that each country would select under unilateral tariff setting were the principals in charge so that the welfare weight is \(\phi_P\). Specifically, for each \(\phi \in \Phi\), we assume that there exists a tariff \(\tau_{opt}(\phi) > 0\) that is non-prohibitive and that maximizes \(\tilde{C}S_x(\tau) + \phi \tilde{P}S_x(\tau) + \tilde{T}R(\tau)\) over non-negative tariffs.\(^{27}\) We then set \(\tau = \tau_{opt}(\phi_P)\). With \(\tau\) set at this value, the pre-negotiation tariffs are then the Nash tariffs, \((\tau, \tau^*) = (\tau, \tau)\), and the foreign country exploits “loopholes” so as to revert to its unilaterally optimal effective tariff, \(\tau\), when possible (i.e., with probability \(\gamma\)).

The home welfare function for the tariff model reflects the uncertainty about the effectiveness of the foreign liberalization for the home export good, and is defined by

\[
\hat{\omega}(\tau, \gamma, \phi) = \tilde{C}S_x(\tau) + \phi \tilde{P}S_x(\tau) + \tilde{T}R(\tau) + \tilde{C}S_y(\tau) + \tilde{P}S_y(\tau) + \gamma[\tilde{C}S_y(\tau) + \tilde{P}S_y(\tau) - \tilde{C}S_y(\tau) - \tilde{P}S_y(\tau)].
\]

The welfare function thus corresponds to the consumer surplus, producer surplus and tariff revenue received by the home country when the parameter \(\phi \in \Phi\) denotes the special welfare weight given to aggregate profit in the import-competing sector and when the home tariff is \(\tau\) and the effective level of foreign protection is reduced to \(\tau\) with probability \(\gamma \in \Gamma\) and takes the higher level of \(\bar{\tau}\) with probability \(1 - \gamma\). The term multiplied by \(\gamma\) thus gives the gain to the home country when the negotiated foreign liberalization is effective.

Finally, we assume that \(\phi_P > \phi_A > 1\), which ensures that both the principal and agent face some political pressure from the import-competing industry and that the principal faces greater pressure.\(^{28}\) This parameter restriction is in line with the traditional view that the executive branch is subject to less sector-level lobbying.

We now rewrite the welfare function in (20) so as to express it in the form given in

\(^{26}\)The negotiation process achieves reciprocity in terms of the (symmetric) negotiated tariff reductions; however, the effective tariff reduction is not reciprocal since the foreign country has a higher effective rate of protection with probability \(\gamma\). With some additional notation, we could alternatively assume that the negotiated tariff reduction is asymmetric, with the foreign country agreeing to a lower tariff so that, in expectation, it offers the same effective level of protection as does the home country.

\(^{27}\)As we confirm in the Appendix, the assumption that \(\tau_{opt}(\phi) > 0\) follows naturally in this “large-country” model, given that \(\phi \geq 1\). The assumption that \(\tau_{opt}(\phi)\) is non-prohibitive is also natural, although this assumption may limit the extent to which \(\phi\) is allowed to exceed unity. The linear-quadratic model that we examine in the next subsection provides an example.

\(^{28}\)When \(\phi_A = 1 = \gamma\), the flexible tariff for the home country (i.e., the tariff that maximizes \(\hat{\omega}(\tau, 1, 1)\)) would be zero in this symmetric model. Correspondingly, in the delegation model, we would have that \(\pi_f(1, 1) = \bar{\pi}(0) = \pi\). The function \(\pi_f(\gamma, \phi_A)\) would thus fall on the boundary of the set \(\Pi\) when \(\phi_A = 1 = \gamma\), which violates Assumption 1. This violation is avoided under our assumption that \(\phi_A > 1\). Alternatively, we could allow \(\phi_A = 1\) and still ensure the interiority of \(\pi_f(\gamma, \phi_A)\) by expanding \(\Upsilon\) to allow for small import subsidies, leading in turn to a value for \(\pi\) that slightly exceeds \(\bar{\pi}(0)\).
We have that \( \hat{\omega}(\tau, \gamma, \phi) = \hat{b}(\tau, \phi) + \gamma \hat{\pi}(\tau) \) and \( \hat{b}(\tau, \phi) = \hat{\beta}(\tau) + \phi \hat{\Lambda}(\tau) \) where

\[
\begin{align*}
\hat{\beta}(\tau) &= \hat{C}S_x(\tau) + \hat{T}R(\tau) + \hat{C}S_y(\tau) + \hat{P}S_y(\tau) \\
\hat{\Lambda}(\tau) &= \hat{P}S_x(\tau) \\
\hat{\pi}(\tau) &= \hat{C}S_y(\tau) + \hat{P}S_y(\tau) - \hat{C}S_y(\tau) - \hat{P}S_y(\tau)
\end{align*}
\]

Thus, for the partial-equilibrium model, \( \hat{b}(\tau, \phi) \) gives the welfare enjoyed by the home country on its import good under preference parameter \( \phi \) when its import tariff is \( \tau \) and on its export good when foreign liberalization turns out to be ineffective. Turning to the components of \( \hat{b}(\tau, \phi) \), we see that \( \hat{\Lambda}(\tau) \) gives the producer surplus for the home import-competing industry while \( \hat{\beta}(\tau) \) captures remaining terms. Finally, we observe that \( \hat{\pi}(\tau) \) captures the welfare gain enjoyed by the home country on its export good when effective foreign liberalization is realized.

In the Appendix, we confirm two key results:

\[
\hat{\pi}'(\tau) < 0 \quad \text{and} \quad \hat{\Lambda}'(\tau) > 0 \quad \text{for all} \quad \tau \in \Upsilon = [0, \tau].
\]

Thus, the partial-equilibrium model satisfies assumptions (16) and (17) given in the previous subsection. Intuitively, our finding that \( \hat{\pi}'(\tau) < 0 \) derives from the embedded reciprocity assumption whereby a lower negotiated home tariff \( \tau \) is paired with a lower negotiated foreign tariff leading under effective foreign liberalization to a higher world price for home’s export good and thus greater export-good welfare for home. As in (18), it now follows that \( \hat{\pi}'(\pi) < 0 \) for \( \pi \in \Pi = [0, \pi] = [\hat{\pi}(\tau), \hat{\pi}(0)] \). As indicated in (21), the partial-equilibrium model of trade also delivers that \( \hat{\Lambda}'(\tau) > 0 \). This finding simply reflects the idea that a higher home tariff raises the local price of the home import good and thus increases the producer surplus in the home import-competing sector.

Using \( \phi_P > \phi_A \) and (21), we may now refer to (19) and conclude that the partial-equilibrium model of trade leads to a delegation model in which the agent has upward bias for all \( \gamma \in \Gamma \), if \( \pi_f(\gamma, \phi_A) \in \Pi \) for all \( \gamma \in \Gamma \). In the Appendix, we show further that \( \pi_f(\gamma, \phi_A) \in \Pi \) indeed holds for all \( \gamma \in \Gamma \). We thus conclude that the partial-equilibrium model of trade and tariff negotiations provides a foundation for the assumption in the delegation model that the agent has upward bias for all \( \gamma \in \Gamma \).

From this foundation, it is possible to appeal to Proposition 5 and Corollary 1 and provide conditions for the optimality of an optimal cap allocation that is interior, or equivalently for the optimality of a tariff floor allocation that is interior, once we confirm that Assumption 1 holds. With regard to Assumption 1, the remaining issue is to provide conditions under which the implied function \( b(\pi, \phi) \) is strictly concave over \( \pi \in \Pi \). In the next subsection, we illustrate that Assumption 1 holds under natural specifications by examining a linear-quadratic version of the partial-equilibrium model. Other examples...
can be considered similarly.\footnote{See Bagwell and Staiger (2005) for further analysis of the linear-quadratic model. Amador and Bagwell (2013) also use the linear-quadratic model to illustrate their findings.}

### 3.3 Linear-Quadratic Example

We now examine a linear-quadratic version of the partial equilibrium model. Specifically, we assume that $u(c) = c - c^2/2$, $Q_x(p_x) = p_x/2$, $Q_y(p_y) = p_y$, $Q_x^*(p_x^*) = p_x^*$, and $Q_y^*(p_y^*) = p_y^*/2$.

In this model, a tariff at or above 1/6 is prohibitive. We may also easily verify that the unilaterally optimal tariff for a country with welfare weight $\phi$ is given by

$$
\tau_{\text{opt}}(\phi) \equiv \frac{8\phi - 5}{4(17 - 2\phi)},
$$

where $\tau_{\text{opt}}(\phi) < 1/6$ provided $\phi < 7/4$. We thus assume that

$$
\frac{7}{4} > \phi_P > \phi_A > 1
$$

where we observe from (22) that $\tau = \tau_{\text{opt}}(\phi_P) \in (0, 1/6)$ given $7/4 > \phi_P > \phi_A > 1$.

For the linear-quadratic model, the home welfare function as defined in (20) takes a simple form:

$$
\hat{\omega}(\tau, \gamma, \phi) = \frac{1}{49}(4\phi + \frac{9}{2} + \tau(8\phi - 5) + 2\tau^2(2\phi - 17))
+ \frac{1}{98}(25 - 6\tau + 18(\tau)^2) + \frac{\gamma}{49}(-3(\tau - \bar{\tau}) + 9((\tau)^2 - (\bar{\tau})^2)).
$$

We can thus write

$$
\hat{\omega}(\tau, \gamma, \phi) = \hat{b}(\tau, \phi) + \gamma \hat{\pi}(\tau) = \hat{\beta}(\tau) + \phi \hat{\Lambda}(\tau) + \gamma \hat{\pi}(\tau),
$$

where

$$
\hat{\beta}(\tau) = \frac{1}{49}(\frac{9}{2} - 5\tau - 34\tau^2) + \frac{1}{98}(25 - 6\tau + 18(\tau)^2)
$$

$$
\hat{\Lambda}(\tau) = \frac{1}{49}(4 + 8\tau + 4\tau^2)
$$

$$
\hat{\pi}(\tau) = \frac{(3\tau - 9(\tau)^2) - (3\tau - 9(\tau)^2)}{49}.
$$
Consistent with (21), we now observe that, for all $\tau \in \Upsilon = [0, \pi]$,

\[
\hat{\Lambda}'(\tau) = \frac{8}{49}(1 + \tau) > 0
\]

\[
\hat{\pi}'(\tau) = \frac{-3(1 - 6\tau)}{49} < 0.
\]

The induced range for $\hat{\pi}(\tau)$ is given by

\[
\Pi = [0, \hat{\pi}] = [\hat{\pi}(\tau), \hat{\pi}(0)],
\]

where $\pi = \hat{\pi}(0) = \frac{(3\pi - 9(\pi)^2)}{49} > 0$.

Since $\hat{\pi}'(\tau) < 0$, we may invert $\pi = \hat{\pi}(\tau)$ while taking the lower root to obtain the inverse relation, $\tau = \hat{\tau}(\pi)$, where

\[
\hat{\tau}(\pi) = \frac{1}{6} - \frac{1}{6}\sqrt{\chi(\pi)}
\]

with

\[
\chi(\pi) = 196\pi + 1 - 12\pi(1 - 3\pi).
\]

Note that $\chi(\pi) > 0$ for all $\pi \in \Pi$, where $\chi(\pi) < 1$ for $\pi < \hat{\pi}(0)$, and $\chi(\pi) = 1$ for $\pi = \hat{\pi}(0)$.

We may now confirm that

\[
\hat{\tau}'(\pi) < 0 < \hat{\tau}''(\pi) \text{ for all } \pi \in \Pi
\]

where $\hat{\tau}(0) = \pi$ and $\hat{\tau}(\pi) = 0$. It follows that $\hat{\tau}(\pi) \in \Upsilon$ for $\pi \in \Pi$.

Referring to (24), we now may express welfare as a function of $\pi$:

\[
\omega(\pi, \gamma, \phi) = \frac{4\phi + \frac{9}{2}}{49} + \frac{25 - 6\pi + 18(\pi)^2}{98} + \hat{\tau}(\pi)(8\phi - 5) - \frac{2(\hat{\tau}(\pi))^2(17 - 2\phi)}{49} + \gamma\pi.
\]

We can thus write

\[
\omega(\pi, \gamma, \phi) = b(\pi, \phi) + \gamma\pi = \beta(\pi) + \phi\Lambda(\pi) + \gamma\pi,
\]

where

\[
\beta(\pi) = \frac{9}{2} + \frac{25 - 6\pi + 18(\pi)^2}{98} - \frac{5\hat{\tau}(\pi)}{49} - \frac{34(\hat{\tau}(\pi))^2}{49}
\]

\[
\Lambda(\pi) = \frac{4}{49} + \frac{8\hat{\tau}(\pi)}{49} + \frac{4(\hat{\tau}(\pi))^2}{49}.
\]

We may now observe that $\Lambda'(\pi) = \frac{8}{49}[1 + \hat{\tau}(\pi)]\hat{\tau}'(\pi) = \hat{\Lambda}'(\pi)\hat{\tau}'(\pi)$. We know, for all
\[ \pi \in \Pi, \ 1 + \hat{\tau}(\pi) > 0 \text{ and } \hat{\tau}'(\pi) < 0; \text{ hence,} \]
\[ \Lambda'(\pi) < 0 \text{ for all } \pi \in \Pi. \]  
(29)

We next characterize \( \pi_f(\gamma, \phi_A) \). In the proof of Lemma 1, we show that
\[ \pi_f(\gamma, \phi_A) = \chi_f(\gamma, \phi_A) + \frac{12(1 - 3\tau)}{196}(1 - \frac{1}{6} - \tau_{opt}(\phi_P)) - \frac{1}{2}(17 - 2\phi_A) - 9\gamma \]  
(30)

where
\[ \chi_f(\gamma, \phi_A) = \left[ \frac{12(17 - 2\phi_A)(1/6 - \tau_{opt}(\phi_P))}{2(17 - 2\phi_A) - 9\gamma} \right]^2. \]  
(31)

Given the restrictions imposed in (23), we may now verify that Assumption 1 holds in this model.

**Lemma 1** Let \( \Pi = [0, \pi], \ \Gamma = [\gamma, \gamma] \) and \( \Psi = \{\phi_P, \phi_A\} \) where \( \pi = \frac{(3\tau - \phi_A^2)}{49}, \ \gamma = 0 < 1 = \gamma, \ 7/4 > \phi_P > \phi_A > 1 \) and \( \tau = \frac{8\phi_P - 5}{4(17 - 2\phi_P)} \). Let \( \beta : \Pi \rightarrow \mathbb{R} \) and \( \Lambda : \Pi \rightarrow \mathbb{R} \) be defined by (27) and (28), respectively, using (25) and (26). Let \( b : \Pi \times \Phi \rightarrow \mathbb{R} \) be defined by \( b(\pi, \phi) = \beta(\pi) + \phi\Lambda(\pi) \), and let \( \pi_f : \Gamma \times \Phi \rightarrow \Pi \) be defined by (30) using (31). Then Assumption 1 holds.

The proof is in the Appendix.

With Assumption 1 holding, we can now apply our propositions. We begin with Proposition 5. The corresponding result is as follows:

**Corollary 2** Let the linear-quadratic model be specified as in Lemma 1. Assume that there exists an optimal cap allocation that is interior, \( \gamma^*_H \in (\gamma, \gamma) \). Assume that the spread between \( \phi_P \) and \( \phi_A \) is sufficiently small in the specific sense that \( 7/4 \geq 2\phi_P - \phi_A \). Then, for \( f'(\gamma) \geq 0, \forall \gamma \in \Gamma \), the cap allocation defined by \( \gamma^*_H \) solves the delegation problems with and without money burning.

Comparing Corollary 2 with Proposition 5, we see that the former dispenses with the assumption that the agent does not exhibit downward bias over a subset of \( \Gamma \). This condition is automatically satisfied in the linear-quadratic model. Specifically, using \( \phi_P > \phi_A \) and referring to (19) and (29), we see that the agent has upward bias at all \( \gamma \in \Gamma \):
\[ \omega_\pi(\pi_f(\gamma, \phi_A), \gamma, \phi_P) = v_\pi(\pi_f(\gamma, \phi_A); \phi_P, \phi_A) < 0, \forall \gamma \in \Gamma. \]  
(32)

Corollary 2 also replaces the assumption that \( \kappa \geq 1/2 \) from Proposition 5 with the assumption that \( 7/4 \geq 2\phi_P - \phi_A \). In the Appendix, we complete the proof of Corollary 2 by providing a closed-form solution for \( \kappa \) in the linear-quadratic model and showing that \( \kappa < 1 \) holds and that \( \kappa \geq 1/2 \) holds if \( 7/4 \geq 2\phi_P - \phi_A \).

Finally, we use (32) to establish the following immediate implication of Corollary 1:
Corollary 3 Let the linear-quadratic model be specified as in Lemma 1. If the level of bias as measured by $\phi_P - \phi_A$ is sufficiently small and the density $f$ satisfies $f'(\gamma) \geq 0$, $\forall \gamma \in \Gamma$, then the delegation problems with and without money burning are both solved by an optimal cap allocation that is interior, $\gamma^*_H \in (\underline{\gamma}, \overline{\gamma})$.

According to Corollary 3, therefore, if the level of bias is sufficiently small, then a non-decreasing density alone is sufficient to guarantee for the linear-quadratic model that the delegation problems with and without money burning are both solved by an optimal cap allocation that is interior.

We now make three further points. First, as mentioned above, the results reported in Corollaries 2 and 3 can be re-interpreted as providing conditions under which a tariff floor allocation is optimal. Second, the conditions given here support optimal delegation contracts that do not involve money burning. Thus, the model presented here does not predict the actual use of costly meetings and reports as a form of money burning. Third, as Amador and Bagwell (2013) show, it is possible to use general optimality conditions to provide sufficient conditions that allow for a density function that decreases over ranges or even over the whole support, provided that the rate of decrease is not too great.

3.4 Alternative Interpretations

In the previous two subsections, we develop and explore a partial-equilibrium model of trade and tariff negotiations in which the agent (the executive branch) has private information about the probability that the negotiated foreign tariff liberalization will be effective. In this subsection, we provide alternative interpretations of the partial-equilibrium model wherein the agent’s private information concerns instead the extent of reciprocity available from the foreign negotiation partner or partners.

We begin with an alternative interpretation in which there is a single foreign country and the agent’s private information concerns the feasible extent of the negotiated foreign tariff liberalization. To link this interpretation to our modeling framework, we modify the trade model in a simple way and allow now that the two countries trade $2N + 1 \geq 3$ products, where $N$ goods are imported by the home country, $N$ goods are imported by the foreign country, and a single numeraire good is traded freely. Each of the $N$ imported goods for the home country is symmetric, so that there are in effect $N$ goods that each have the demand and supply functions attributed to good $x$ in the three-good model considered in the previous two subsections. Similarly, each of the $N$ imported goods for the foreign country is symmetric and has the same attributes as good $y$ in the three-good

---

30See Corollary 2, part (ii) in Amador and Bagwell (2013). For a generalization of this argument, see Lemma 1 of Amador and Bagwell (2018).
model. The underlying quasi-linear utility function for consumers in each country then entails the summation of $2N$ identical functions plus the numeraire term,

$$\sum_{i=x,y} \sum_{j=1,...,N} u(c_{ij}) + c_n,$$

where good $n$ is again the numeraire good.

Since each home import good is symmetric, we assume that the home country applies the same import tariff, $\tau$, to each of its import goods. Likewise, we assume that the foreign country applies the same import tariff, $\tau^*$, to each of its import goods. For each import good within a given country, therefore, the same consumer surplus, producer surplus and tariff revenue is received. Similarly, for each export good within a given country, the same consumer surplus and producer surplus is enjoyed.

Turning to the negotiation, we assume again that the home and foreign country begin with a symmetric status quo tariff, $\overline{\tau}$, and negotiate a symmetric tariff value, $\tau = \tau^* \in \Upsilon$, selected by the agent (the executive branch of the home country). The home country tariff selection, however, is constrained to belong to the permissible set of tariffs as determined by the principal (the legislative branch of the home country). The novel feature that we introduce here is that the agent has private information about the fraction of goods, $\gamma \in \Gamma$, for which the foreign country agrees to reduce its tariff from the status quo level, $\overline{\tau}$, to the negotiated level, $\tau$. Naturally, a lower negotiated tariff $\tau$ is more attractive for the home country when the foreign country is able to accept the tariff reduction for a larger fraction of its import goods. In analogy with (20), we may represent the home-country welfare function as

$$\tilde{\Omega}(\tau, \gamma, \phi) = N \{ \tilde{C}S_x(\tau) + \phi \tilde{P}S_x(\tau) + \tilde{P}R(\tau) + \tilde{C}S_y(\overline{\tau}) + \tilde{P}S_y(\overline{\tau})$$

$$+ \gamma [\tilde{C}S_y(\tau) + \tilde{P}S_y(\tau) - \tilde{C}S_y(\overline{\tau}) - \tilde{P}S_y(\overline{\tau})] \}$$

or equivalently as

$$\hat{\Omega}(\tau, \gamma, \phi) = N \cdot \tilde{\omega}(\tau, \gamma, \phi).$$

Thus, the welfare functions of the principal ($\phi = \phi_P$) and agent ($\phi = \phi_A$) are the same as those in (20), up to a common scaling term $N$. It follows that all of the results above carry over exactly as stated when the welfare functions in the underlying tariff model take the form given in (34).\footnote{Indeed, if we think of each country as importing a unit mass of goods (while also trading the numeraire good), then we can set $N = 1$ so that $\hat{\Omega}(\tau, \gamma, \phi) = \tilde{\omega}(\tau, \gamma, \phi)$.}

Our second alternative interpretation is similar but allows for multiple foreign countries. We retain the modification under which the home country trades $2N + 1 \geq 3$ products, but we now assume that there are $N$ foreign countries, each of which exports...
one of the home country’s (non-numeraire) import goods. The model is otherwise as above. We then interpret the home country as having private information as to the fraction $\gamma$ of foreign countries for which a tariff reduction from $\tau$ to $\tau$ is feasible (or effective).\footnote{In addition to the reasons given in Section 3.2 for our primary interpretation of the model, negotiated foreign tariff liberalization may be ineffective (or at least less effective) here for countries for which the pre-negotiation applied tariff is already at or close to the level $\tau$. If a fraction $\gamma$ of foreign countries have such “binding overhang,” then the value to the home country of a negotiated reduction in the tariff binding for these countries from $\tau$ to $\tau$ may be less significant.} Once again, the welfare functions of the principal ($\phi = \phi_P$) and agent ($\phi = \phi_A$) can be represented as in (33) and (34), so that all of the results above again carry over exactly as stated.

Hence, under these alternative interpretations, the legislative branch gives a tariff floor to the executive branch in order to limit the expression of the executive branch’s low-tariff bias (when $\phi_P > \phi_A$) while at the same time granting the executive branch some discretion in view of its superior information about the extent of negotiated tariff liberalization that is available from the foreign trading partner or partners. In this way, we offer alternative interpretations of tariff floors that build from the idea that during the course of negotiations the executive branch may acquire a deeper understanding of the extent of reciprocity available from its foreign partner or partners.

\section{Conclusion}

This paper explores the relationship between the theory of optimal delegation and the delegation of tariff-negotiation authority under U.S. legislation following the Reciprocal Trade Agreements Act (RTAA) of 1934.

To this end, we specify a set of general welfare functions for the principal and agent within which the extent of bias may be easily varied. The model is well-suited for application to delegation settings in which the principal and agent have related but distinct preferences. The framework also fits within the general delegation framework considered by Amador and Bagwell (2013), so that the implications of the general conditions developed in that paper can be used. For the delegation problems with and without money burning, we then provide general conditions under which optimal delegation takes the form of an optimal cap that is interior. These conditions are easily checked when the extent of bias is small.

To illustrate the application of the framework, we then consider the delegation of tariff-negotiation authority under U.S. legislation following the RTAA. With the legislative and executive branches playing the respective roles of principal and agent, we follow traditional arguments and assume that both branches are sensitive to pressure from import-competing interests but that the executive branch is less so. The executive branch is thus more...
attracted to lower tariffs than is the legislative branch. We also assume that the executive branch acquires relevant private (or at least non-verifiable) information perhaps during the course of the negotiation with the foreign country. While there are many possible specifications that might be considered, we assume that the executive branch has private information about the probable effectiveness or the extent of the negotiated foreign tariff liberalization. We then apply the implications of delegation theory as developed in the first part of the paper and provide conditions under which the optimal form of trade-policy authority for the negotiation of reciprocal tariff agreements entails a tariff floor with no money burning used. We argue that this form of delegation is consistent with important features of the RTAA and subsequent legislation.

The paper suggests several promising extensions. It would be interesting to develop other applied models of the negotiation process, perhaps featuring alternative sources of private information. As one example, future research might consider a two-good, general-equilibrium model in which reciprocity holds in a strict form and fixes the world price but the principal does not observe (or cannot verify) the prevailing world price at the time of negotiation. It would also be interesting to extend the analysis to include “escape-clause” provisions as allowed under U.S. legislation and GATT/WTO rules under which negotiated tariffs may be raised if serious injury to the import-competing industry is determined. Finally, our trade-policy application demonstrates that the framework developed by Amador and Bagwell (2013) directly includes a range of applications in which private information concerns the probability \( \gamma \) that an outcome will be realized. We expect that other applications can be similarly developed building from this observation.

33 Bagwell and Staiger (1999, 2018) consider related models of negotiation, but they do not study a delegation game that arises across domestic political branches.

34 For discussion of escape-clause provisions under the RTAA, see Leddy and Norwood (1963). See Bagwell and Staiger (1990, 2005) and Bond and Bheskar (2016, 2017) for interpretations of the escape-clause provisions in GATT/WTO rules using, respectively, repeated-game theory and delegation theory with the possibility of costly state verification. Halac and Yared (2019) develop a general theoretical framework for considering models of the latter kind.
5 Appendix

Proof of Proposition 1: The optimal cap allocation is defined by the value for $\gamma_H$ that maximizes the following objective:

$$
\int_{\gamma}^{\gamma_H} \omega(\pi_f(\gamma, \phi_A), \gamma, \phi_P) dF(\gamma) + \int_{\gamma_H}^{\gamma} \omega(\pi_f(\gamma_H, \phi_A), \gamma_H, \phi_P) dF(\gamma).
$$

The first-order condition for maximizing the objective is

$$
Z(\gamma_H; \phi_A, \phi_P) \equiv \frac{\partial \pi_f(\gamma_H, \phi_A)}{\partial \gamma} \int_{\gamma_H}^{\gamma} \omega(\pi_f(\gamma_H, \phi_A), \gamma_H, \phi_P) dF(\gamma) = 0,
$$

(35)

where we recall from Assumption 1 that $\frac{\partial \pi_f(\gamma, \phi_A)}{\partial \gamma} > 0$. Notice that first derivative when evaluated at $\gamma_H = \gamma$ is zero:

$$
Z(\gamma; \phi_A, \phi_P) = 0.
$$

Now consider the second derivative of the objective. It is

$$
Z_{\gamma_H}(\gamma_H; \phi_A, \phi_P) = \frac{\partial^2 \pi_f(\gamma_H, \phi_A)}{\partial \gamma^2} \int_{\gamma_H}^{\gamma} \omega(\pi_f(\gamma_H, \phi_A), \gamma_H, \phi_P) dF(\gamma)
$$

(36)

$$
+ \left( \frac{\partial \pi_f(\gamma_H, \phi_A)}{\partial \gamma} \right)^2 \int_{\gamma_H}^{\gamma} \omega(\pi_f(\gamma_H, \phi_A), \gamma_H, \phi_P) dF(\gamma)
$$

$$
- \frac{\partial \pi_f(\gamma_H, \phi_A)}{\partial \gamma} \omega(\pi_f(\gamma_H, \phi_A), \gamma_H, \phi_P) f(\gamma_H).
$$

When we evaluate the second derivative at $\gamma_H = \gamma$, we get

$$
Z_{\gamma_H}(\gamma; \phi_A, \phi_P) = - \frac{\partial \pi_f(\gamma, \phi_A)}{\partial \gamma} \omega(\pi_f(\gamma, \phi_A), \gamma, \phi_P) f(\gamma) > 0,
$$

which follows from Assumption 1 and (6).

Summarizing, at the upper bound, $\gamma_H = \gamma$, we have that

$$
Z(\gamma; \phi_A, \phi_P) = 0 < Z_{\gamma_H}(\gamma; \phi_A, \phi_P),
$$

so that $Z(\gamma_H; \phi_A, \phi_P)$ approaches zero from below as $\gamma_H$ approaches $\gamma$.
Now consider the lower bound, $\gamma_H = \gamma$. We have that
\[
Z(\gamma; \phi_A, \phi_P) = \frac{\partial \pi_f(\gamma, \phi_A)}{\partial \gamma} \int_{\gamma}^{\gamma_H} \omega_\pi(\pi_f(\gamma, \phi_A), \tilde{\gamma}, \phi_P) dF(\tilde{\gamma})
\]
\[
= \frac{\partial \pi_f(\gamma, \phi_A)}{\partial \gamma} \int_{\gamma}^{\gamma_H} (\omega_\pi(\pi_f(\gamma, \phi_A), \gamma_H, \phi_P) + \tilde{\gamma} - \gamma) dF(\tilde{\gamma}),
\]
since $\omega_\pi(\pi, \gamma, \phi) = b_\pi(\pi, \phi) + \gamma$. Thus,
\[
Z(\gamma; \phi_A, \phi_P) = \frac{\partial \pi_f(\gamma, \phi_A)}{\partial \gamma} \left[ \omega_\pi(\pi_f(\gamma, \phi_A), \gamma, \phi_P) + E(\gamma) - \gamma \right]
\]
\[
> 0,
\]
where the inequality follows from Assumption 1 and (7).

Pulling these themes together, under (6) and (7), the first derivative of the objective is positive when evaluated with $\gamma_H = \gamma$, and the first derivative is zero and strictly increasing at $\gamma_H = \gamma$. Given that the first derivative is continuous under Assumption 1, there then must exist $\gamma^*_H \in (\gamma, \gamma)$ at which the first-order condition holds with $Z(\gamma^*_H; \phi_A, \phi_P) = 0$ and the objective is maximized. QED

Proof of Proposition 3: Part (i) follows immediately from (4) and (11), and part (iii) follows immediately from (11). For part (ii), the supposition implies that there exists $\gamma^*_H \in (\gamma, \gamma)$ at which the first-order condition for an optimal cap allocation is satisfied. Referring to (35) from the proof of Proposition 1 and using Assumption 1, we thus have that
\[
0 = \int_{\gamma_H}^{\gamma} \omega_\pi(\pi_f(\gamma^*_H, \phi_A), \tilde{\gamma}, \phi_P) dF(\tilde{\gamma})
\]
\[
= \int_{\gamma_H}^{\gamma} [\omega_\pi(\pi_f(\gamma^*_H, \phi_A), \gamma^*_H, \phi_P) + \tilde{\gamma} - \gamma^*_H] dF(\tilde{\gamma})
\]
\[
= [\phi_P - \phi_A] \Lambda'(\pi_f(\gamma^*_H, \phi_A)) - \gamma^*_H] [1 - F(\gamma^*_H)] + \int_{\gamma_H}^{\gamma} \tilde{\gamma} dF(\tilde{\gamma})
\]
\[
= [v_\pi(\pi_f(\gamma^*_H, \phi_A); \phi_P, \phi_A) - \gamma^*_H] [1 - F(\gamma^*_H)] + \int_{\gamma_H}^{\gamma} \tilde{\gamma} dF(\tilde{\gamma})
\]
\[
= [1 - F(\gamma^*_H)] [v_\pi(\pi_f(\gamma^*_H, \phi_A); \phi_P, \phi_A) - \gamma^*_H + E(\gamma | \gamma \geq \gamma^*_H)]
\]
where the second equality uses that $\omega_\pi(\pi, \gamma, \phi) = b_\pi(\pi, \phi) + \gamma$, the third equality uses (5), the fourth equality follows from (11) and the final equality follows directly. Given that
\( \gamma_H^* \in (\gamma, \bar{\gamma}) \), it now follows that (12) holds. QED

Properties of Partial-Equilibrium Model in Section 3.2: We provide here further details regarding the properties of the partial-equilibrium model in Section 3.2.\(^{35}\) We also establish (21).

As indicated in the text, we consider a two-country, three-good partial-equilibrium model, in which the home country imports good \( x \) and exports good \( y \), with the numeraire good \( n \) freely traded. The demand side in each country is determined by a common quasi-linear utility function, \( u(c_x) + u(c_y) + c_n \), where \( u \) is strictly increasing, strictly concave and thrice continuously differentiable. The supply side is described by perfectly competitive sectors, with the supply functions for goods \( x \) and \( y \) in the home (foreign) country taking the form \( Q_x(p_x) \) and \( Q_y(p_y) \) (\( Q_x^*(p_x^*) \) and \( Q_y^*(p_y^*) \)), where \( p_x \) and \( p_y \) (\( p_x^* \) and \( p_y^* \)) are the local prices of goods \( x \) and \( y \) in the home (foreign) country. For any common price level \( p \), we assume the supply functions take mirror image forms: \( Q_x(p) = Q_y^*(p) \) and \( Q_y(p) = Q_x^*(p) \). For any price that elicits strictly positive supply, we assume that the associated supply function is strictly increasing and twice continuously differentiable. Finally, for any price \( p \) such that there is strictly positive world supply, we assume that \( Q_x(p) < Q_x^*(p) \). This ensures that the home country imports (exports) good \( x \) (\( y \)) under global free trade.

As is standard, we assume that the numeraire good is produced in each country under constant returns to scale. Labor is the only factor, and the supply of labor is perfectly inelastic. The numeraire good is produced and consumed in each country, and we set the wage and price of the numeraire good equal to one. Thus, the prices of goods \( x \) and \( y \) in each country are relative to the unitary price of the numeraire good \( n \).

We start with good \( x \). Using \( z_x \) to denote the volume of trade across the two countries in good \( x \), we represent the optimization problem for home consumers as

\[
u'(Q_x(p_x) + z_x) = p_x \Rightarrow p_x = P_x(z_x). \tag{37}\]

Likewise, the optimization problem for foreign consumers is represented as

\[
u'(Q_x^*(p_x^*) - z_x) = p_x^* \Rightarrow p_x^* = P_x^*(z_x). \tag{38}\]

For \( z_x > 0 \), we note that (37) implies

\[
P_x'(z_x) = \frac{-u''(Q_x(p_x) + z_x)}{u''(Q_x(p_x) + z_x)Q_x'(p_x)} - 1 < 0 \tag{39}\]

\(^{35}\)Our discussion here is similar in places to that provided in Amador and Bagwell (2013a).
and (38) likewise implies

\[ P_x''(z_x) = \frac{u''(Q^*_x(p^*_x) - z_x)}{u''(Q^*_x(p^*_x) - z_x)Q''^*_x(p^*_x) - 1} > 0, \]  

(40)

where the strict inequalities in (39) and (40) follow from the strict concavity of \( u \) and the fact that the supply functions are strictly increasing.

Each country has available a specific import tariff, denoted as \( \tau \) (\( \tau^* \)) for the home (foreign) country. We assume that \( \tau \in [0, \bar{\tau}] = \Upsilon \) and that likewise \( \tau^* \in \Upsilon \), where \( \bar{\tau} > 0 \). Using (37) and (38), we note that prices are further related via the requirement that

\[ \tau = P_x(z_x) - P^*_x(z_x). \]  

(41)

Using (41), we can represent the trade volume as a function of the tariff, \( z_x = z_x(\tau) \), and derive that

\[ z'_x(\tau) = \frac{1}{P''_x(z_x) - P'^*_x(z_x)} < 0, \]  

(42)

where we use (39) and (40). Of course, we can also think of choosing the trade volume with the tariff then implied by (41), so that \( \tau = \tau(z_x) \). For this functional relationship, (39), (40) and (41) give that

\[ \tau'(z_x) = P'_x(z_x) - P'^*_x(z_x) < 0. \]  

(43)

For a given trade volume \( z_x \), the home and foreign producer surpluses for good \( x \) are respectively given by

\[ \tilde{PS}_x(z_x) = \int_{P_x}^{P_x(z_x)} Q_x(p)dp \]  

(44)

and

\[ \tilde{PS}^*_x(z_x) = \int_{P^*_x}^{P^*_x(z_x)} Q^*_x(p)dp, \]  

(45)

where \( P_x \) and \( P^*_x \) denote the highest prices below which supply of good \( x \) is zero in the home and foreign countries, respectively. We observe that \( \tilde{PS}_x(z_x) \) is the producer surplus in the home import-competing industry while \( \tilde{PS}^*_x(z_x) \) gives the producer surplus in the foreign export sector.

For a given trade volume \( z_x \), we next represent the home and foreign consumer surpluses for good \( x \), respectively, as

\[ \tilde{CS}_x(z_x) = u(Q_x(P_x(z_x)) + z_x) - P_x(z_x) \cdot (Q_x(P_x(z_x)) + z_x) \]  

(46)
\[ \tilde{CS}_x(z_x) = u(Q_x(P_x(z_x)) - z_x) - P_x(z_x) \cdot (Q_x(P_x(z_x)) - z_x). \]  

(47)

Next, for a given trade volume \( z_x \), we may represent the home tariff revenue as
\[ \tilde{TR}(z_x) = (P_x(z_x) - P_x^*(z_x)) \cdot z_x, \]  

(48)

where we recall from (41) that \( \tau = P_x(z_x) - P_x^*(z_x) \).

Finally, given the symmetry of the model, we may easily represent the home-country producer and consumer surplus terms for its export good \( y \). For a given trade volume \( z_y > 0 \), we may show in analogy with (39) and (40) that
\[ P_y''(z_y) = \frac{-u''(Q_y(P_y) + z_y)}{u''(Q_y(P_y) + z_y)Q_y'(P_y) - 1} < 0 \]  

(49)

and
\[ P_y'(z_y) = \frac{u''(Q_y(P_y) - z_y)}{u''(Q_y(P_y) - z_y)Q_y'(P_y) - 1} > 0 \]  

(50)

In analogy with (41), we have that
\[ \tau^* = P_y^*(z_y) - P_y(z_y); \]  

(51)

hence, we can represent the trade volume as a function of the tariff, \( z_y = z_y(\tau^*) \), and derive using (49), (50) and (51) that
\[ z_y'({\tau^*}) = \frac{1}{P_y''(z_y) - P_y'(z_y)} < 0. \]  

(52)

Once again, we can also think of choosing the trade volume with the tariff then implied, so that \( \tau^* = \tau^*(z_y) \). For this functional relationship, we have from (51) that
\[ \tau^{**}(z_y) = P_y''(z_y) - P_y'(z_y) < 0, \]  

(53)

where the inequality uses (49) and (50).

For a given trade volume \( z_y \), the home producer and consumer surplus terms for good \( y \) may now be respectively defined in analogy with (45) and (47) as
\[ \tilde{PS}_y(z_y) = \int_{P_y(z_y)}^{P_y'(z_y)} Q_y(p)dp \]  

(54)

and
\[ \tilde{CS}_y(z_y) = u(Q_y(P_y(z_y)) - z_y) - P_y(z_y) \cdot (Q_y(P_y(z_y)) - z_y), \]  

(55)
where \( p_y \) denotes the highest price below which supply of good \( y \) is zero in the home country.

Using the functional relationships \( z_x = z_x(\tau) \) and \( z_y = z_y(\tau^*) \) as defined from (41) and (51), respectively, we now express the various home-country surplus and revenue expressions as functions of \( \tau \) and \( \tau^* \):

\[
\begin{align*}
\hat{PS}_x(\tau) &= \tilde{PS}_x(z_x(\tau)) \\
\hat{CS}_x(\tau) &= \tilde{CS}_x(z_x(\tau)) \\
\hat{TR}(\tau) &= \tilde{TR}(z_x(\tau)) \\
\hat{PS}_y(\tau^*) &= \tilde{PS}_y(z_y(\tau^*)) \\
\hat{CS}_y(\tau^*) &= \tilde{CS}_y(z_y(\tau^*)).
\end{align*}
\]

With (56), we now have all the ingredients for \( \hat{\omega}(\tau, \gamma, \phi) \) as defined in (20).

Referring to (56) and using (37), (41), (42), (44), (46), (48), (50), (52), (54) and (55) we derive that

\[
\begin{align*}
\hat{CS}_x'(\tau) &= -P_x'(z_x(\tau)) \cdot [Q_x(P_x(z_x(\tau))) + z_x(\tau)] \cdot z_x'(\tau) < 0 \\
\hat{PS}_x'(\tau) &= P_x'(z_x(\tau)) \cdot Q_x(P_x(z_x(\tau))) \cdot z_x'(\tau) > 0 \\
\hat{TR}'(\tau) &= z_x(\tau) + \tau \cdot z_x'(\tau) \\
\hat{CS}_y'(\tau^*) &= -P_y'(z_y(\tau^*)) \cdot [Q_y(P_y(z_y(\tau^*))) - z_y(\tau^*)] \cdot z_y'(\tau^*) > 0 \\
\hat{PS}_y'(\tau^*) &= P_y'(z_y(\tau^*)) \cdot Q_y(P_y(z_y(\tau^*))) \cdot z_y'(\tau^*) < 0
\end{align*}
\]

and where \( \hat{CS}_x'(\tau) < 0 < \hat{PS}_x'(\tau) \) follows from (39) and (42) and where \( \hat{CS}_y'(\tau^*) > 0 > \hat{PS}_y'(\tau^*) \) follows from (50) and (52).

Recall now our assumption that, for each \( \phi \in \Psi = \{ \phi_P, \phi_A \} \), there exists a tariff \( \tau_{\text{opt}}(\phi) > 0 \) that is non-prohibitive and maximizes \( \hat{CS}_x(\tau) + \phi \cdot \hat{PS}_x(\tau) + \hat{TR}(\tau) \) over non-negative tariffs. The corresponding first-order condition is given by

\[
\hat{CS}_x'(\tau) + \phi \cdot \hat{PS}_x'(\tau) + \hat{TR}'(\tau) = 0.
\]
Given \( \phi \geq 1 \), we see that the first-order condition cannot hold at free trade, since
\[
\tilde{C}S_x'(0) + \phi \cdot \tilde{P}S_x'(0) + \tilde{T}R'(0) \\
\geq \tilde{C}S_x'(0) + \tilde{P}S_x'(0) + \tilde{T}R'(0) \\
= [1 - P_x'(z_x(0)) \cdot z_x'(0)] \cdot z_x(0) \\
= -P_x''(z_x(0)) \\
> 0,
\]
where the first inequality uses \( \tilde{P}S_x'(0) > 0 \) from (57) and \( \phi \geq 1 \), the first equality follows from (57), the second equality uses (42), and the final inequality uses (39), (40) and the assumption of a positive trade volume at free trade. We conclude that \( \tau_{opt}(\phi) > 0 \) is indeed a natural assumption in this “large-country” model. Our assumption that \( \tau_{opt}(\phi_P) = \tau > 0 \) is thus well motivated.

With the model now developed in detail, we proceed to establish (21). First, we recall from the text that \( \hat{\Lambda}(\tau) = \tilde{P}S_x(\tau) \); thus, \( \hat{\Lambda}'(\tau) = \tilde{P}S_x'(\tau) > 0 \) follows from (57). Since \( \tilde{P}S_x'(\tau) > 0 \) holds for all \( z_x > 0 \), and since trade volumes are strictly positive for all \( \tau \in \Upsilon \), we have that \( \hat{\Lambda}'(\tau) > 0 \) for all \( \tau \in \Upsilon \). Next, we recall from the text that \( \hat{\pi}(\tau) = \tilde{C}S_y(\tau) + \tilde{P}S_y(\tau) - \tilde{C}S_y'(\tau) - \tilde{P}S_y'(\tau) \). Thus, using (57), we have that
\[
\hat{\pi}'(\tau) = \tilde{C}S_y'(\tau) + \tilde{P}S_y'(\tau) = P_y'(z_y(\tau)) \cdot z_y(\tau) \cdot z_y'(\tau) < 0,
\]
where the inequality follows from (50) and (52). Since \( \hat{\pi}'(\tau) < 0 \) holds for \( z_y(\tau) > 0 \), and since trade volumes are strictly positive for all \( \tau \in \Upsilon \), we have that \( \hat{\pi}'(\tau) < 0 \) for all \( \tau \in \Upsilon \). We have thus established (21).

**Proof of Lemma 1**: First, given that \( \hat{\tau}(\pi) \in \Upsilon \) for \( \pi \in \Pi \) and that \( \hat{\tau}(\pi) \) is twice continuously differentiable, we may easily verify that the functions \( \beta : \Pi \to \Re \) and \( \Lambda : \Pi \to \Re \) are bounded and twice continuously differentiable on \( \Pi \). Second, calculations confirm that \( b(\pi, \phi) = \beta(\pi) + \phi \Lambda(\pi) \) satisfies
\[
\frac{\partial^2 b(\pi, \phi)}{\partial \pi^2} = \frac{392(x(\pi))^{-3/2}(17 - 2\phi)}{3} [\tau_{opt}(\phi_P) - \frac{1}{6}] < 0, \text{ for all } \pi \in \Pi.
\]
Thus, for any \( \phi \in \Phi \), the function \( b : \Pi \times \Phi \to \Re \) is strictly concave on \( \Pi \).

Third, it now follows that \( \gamma \pi + b(\pi, \phi_A) \) is strictly concave on \( \Pi \). We thus examine the first-order condition, \( \gamma + \frac{\partial b(\pi, \phi_A)}{\partial \pi} = 0 \), and find that it is satisfied by \( \pi = \pi_f(\gamma, \phi_A) \) if the induced value for \( \chi(\pi) \) satisfies
\[
\chi_f(\gamma, \phi_A) \equiv \chi(\pi_f(\gamma, \phi_A)) = \left[ \frac{12(17 - 2\phi_A)(1/6 - \tau_{opt}(\phi_P))}{2(17 - 2\phi_A) - 9\gamma} \right]^2.
\]
Since by definition $\chi(\pi_f(\gamma, \phi_A)) = 196\pi_f(\gamma, \phi_A) + 1 - 12\pi(1 - 3\pi)$, we can solve for $\pi_f(\gamma, \phi_A)$, obtaining
\[
\pi_f(\gamma, \phi_A) = \frac{\chi_f(\gamma, \phi_A) + 12\pi(1 - 3\pi) - 1}{196},
\]
where we recall that $\tau = \tau_{\text{opt}}(\phi_P)$ under (23). We observe that $\pi_f(\gamma, \phi_A)$ is twice differentiable on $\Gamma$ and that $\frac{\partial \pi_f(\gamma, \phi_A)}{\partial \gamma} > 0$ for all $\gamma \in \Gamma$. We find that $\pi_f(\gamma, \phi_A) < \bar{\pi} = \pi(0) = \frac{(3\pi - 9(\pi)^2)}{49}$ holds for all $\gamma \in \Gamma$ if and only if $24\phi_A - 15 - 9\gamma > 0$ for all $\gamma \in \Gamma$, which in turn holds under (23).\(^{36}\) Using $\bar{\pi} = \tau_{\text{opt}}(\phi_P)$, we find that $\pi_f(\gamma, \phi_A) > 0$ holds under (23) if and only if
\[
2(17 - 2\phi_A)(\tau_{\text{opt}}(\phi_P) - \tau_{\text{opt}}(\phi_A)) > -9\gamma\left(\frac{1}{6} - \tau_{\text{opt}}(\phi_P)\right),
\]
which in turn holds for all $\gamma \in \Gamma$ under (23) since the function $\tau_{\text{opt}}(\phi)$ is strictly increasing, $\phi_P > \phi_A$, $\frac{1}{6} > \tau_{\text{opt}}(\phi_P)$ and $\gamma \in [0, 1]$. We conclude that there exists a function $\pi_f : \Gamma \times \Phi \to \Pi$ such that, for $\phi = \phi_A$ and for all $\gamma \in \Gamma$, $\pi_f(\gamma, \phi_A) \in (0, \bar{\pi})$, $\pi_f(\gamma, \phi_A)$ is twice differentiable on $\Gamma$, $\frac{\partial \pi_f(\gamma, \phi_A)}{\partial \gamma} > 0$ and $\pi_f(\gamma, \phi_A)$ is the unique maximizer of $\gamma\pi + b(\pi, \phi_A)$ over $\pi \in \Pi$. QED

**Proof of Corollary 2:** We find that $\Lambda''(\pi) = \frac{3}{217\pi''(\pi)} > 0$ for all $\pi \in \Pi$. Given this inequality and that $\phi_P > \phi_A$ from (23), we may conclude from Proposition 2 that $\kappa < 1$. Indeed, we can calculate a closed-form solution for $\kappa$. We find that
\[
\kappa = \frac{\frac{7}{4} - \phi_P}{\frac{7}{4} - \phi_A} \in (0, 1),
\]
where $1 > \kappa > 0$ follows easily from our assumption that $\frac{7}{4} > \phi_P > \phi_A$. It is also of interest to assess $\kappa - 1/2$. We find that $\kappa \geq 1/2$ holds if the spread between $\phi_P$ and $\phi_A$ is sufficiently small in the specific sense that $7/4 \geq 2\phi_P - \phi_A$. By contrast, if the spread between $\phi_P$ and $\phi_A$ is sufficiently large in the specific sense that $7/4 < 2\phi_P - \phi_A$, then $\kappa < 1/2$ obtains. QED

\(^{36}\)When $\phi_A = 1 = \gamma$, we find that $24\phi_A - 15 - 9\gamma = 0$ and so $\pi_f(1, 1) = \bar{\pi} = \pi(0)$. The function $\pi_f(\gamma, \phi_A)$ thus falls on the boundary of the set $\Pi$ when $\phi_A = 1 = \gamma$. We note, though, that our restriction (23) imposes $\phi_A > 1$ and thus does not allow this specification. Alternatively, we could allow $\phi_A = 1$ and still ensure the interiority of $\pi_f(\gamma, \phi_A)$ by expanding $\bar{\gamma}$ to allow for small import subsidies, leading in turn to a value for $\bar{\gamma}$ that slightly exceeds $\bar{\pi}(0)$.
References


