Trade Policy under Monopolistic Competition with Heterogeneous Firms and Quasi-linear CES Preferences*

Preliminary Draft

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Abstract

We analyze trade policy in a symmetric, two-country, two-sector model, where one sector produces differentiated varieties under monopolistic competition with heterogeneous firms while the other sector is competitive and produces a freely traded and homogeneous outside good. Consumer preferences are represented by a quasi-linear CES utility function; thus, differentiated varieties are aggregated according to a CES preference function, and the outside good enters the utility function in a linear and additive fashion. Countries select ad valorem import and export tariffs (or subsidies), where the goal of each country is to maximize its national welfare. We characterize unilateral policy interventions that raise the welfare of the intervening country and harm its trading partner, efficient trade policies that maximize the joint welfare of the two countries, and Nash trade policies. Among other findings, we show that, starting at global free trade, the unilateral introduction of a small export subsidy is beneficial to the intervening country and raises joint welfare, even though the trading partner is harmed. In this sense, the model fails to provide an efficiency-based rationale for a prohibition on the use of export subsidies.

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1 Introduction

We analyze trade policy in a symmetric, two-country, two-sector model, where one sector produces differentiated varieties under monopolistic competition with heterogeneous firms while the other sector is competitive and produces a freely traded and homogeneous outside good. Consumer preferences are represented by a quasi-linear CES utility function. Countries select ad valorem import and export tariffs (or subsidies), where the goal of each country is to maximize its national welfare. We characterize unilateral policy interventions that raise the welfare of the intervening country and harm its trading partner, efficient trade policies that maximize the joint welfare of the two countries, and Nash trade policies.

We assume that trade policies precede the entry, production and pricing decisions of firms and the purchase decisions of consumers. In particular, after trade policies are selected and observed, firms enter the differentiated sector in each country. As in Melitz's (2003) model, each entering firm pays a fixed cost to discover its productivity and then decides whether to produce in the domestic and export markets, respectively, where domestic and export sales involve distinct fixed costs. Export sales are subject to a trade cost as well as tariffs. The markets are segmented, and surviving firms in any given market set prices under conditions of monopolistic competition. Consumers then make their purchase decisions, where any tariff revenue is distributed to consumers in lump sum fashion. Firms have rational expectations at the time of entry, and enter until expected profits are zero.

In addition to the differentiated sector, the model includes a second sector with a homogeneous outside good. The outside good is supplied under conditions of perfect competition at constant unit cost and is freely traded with no trade costs. The consumer utility function takes a quasi-linear CES form; thus, differentiated varieties are aggregated according to a CES preference function, and the outside good enters the utility function in a linear and additive fashion. As is standard, trade in the outside good is residual in nature and ensures trade balance. We focus on the differentiated sector and interpret the outside-good sector as capturing the rest of the economy.[1]

An important feature of the model is that the two sectors are governed by different markups: the outside sector has no markup while the differentiated sector has a positive markup that is constant across firms. As a consequence, the level of entry across sectors may be distorted, creating a corrective role for policy. We focus here on trade policies and characterize such policies under the assumption that domestic policies are unavailable. In this setting, the determination of trade policies is influenced by entry distortions.

Formally, we characterize three driving forces in the model. First, we characterize a

[1] Our analysis can be generalized to allow for multiple differentiated sectors. See Helpman and Itskhoki (2010) for a related perspective.

selection effect: an increase in the overall barrier to trade from the foreign country to the home country, whether achieved via an increase in the home country's import tariff or an increase in the foreign country's export tariff, raises the cut-off productivity level for domestic sales in the home country and lowers the cut-off productivity level for domestic sales in the foreign country. Second, the selection effect and associated entry behavior generate a *Metzler paradox*: an increase in the overall barrier to trade from the foreign country to the home country, whether achieved via an increase in the home country's import tariff or an increase in the foreign country's export tariff, lowers the price index in the home country and raises the price index in the foreign country. Thus, for example, if the home country raises its import tariff, then, once entry adjusts so as to restore zero expected profits in both markets, the price index in the home country falls. Third, we characterize the *entry-externality effect* that leads the market to provide a socially inefficient level of entry. To show that the market in fact always under-provides entry, we consider a closed-economy benchmark version of the model and show that a social planner would always gain by subsidizing the fixed cost of entry for firms.

With the driving forces thus identified, we next provide several results about trade policy. For our first set of results, we assume that countries start at global free trade and consider small trade-policy interventions. We show that a country always gains when it introduces a small import tariff, since it gets tariff revenue and generates a lower price index for its consumers under the Metzler paradox. A country also always gains in this model from the introduction of a small export subsidy. Under the Metzler paradox, a small export subsidy generates a lower price index for the intervening country; however, a small export subsidy also entails an expense that lowers consumer income. Intuitively, under the entry-externality effect, a small export subsidy raises global welfare, and the intervening country gets a larger share of global welfare. Finally, we show that a country can also always gain by introducing both a small import tariff and a small export tariff, where the tariffs are introduced so as to leave the cut-off productivity level for domestic sales in the intervening country constant. Such an intervention leaves the price index constant for the intervening country and delivers tariff revenue. We further show that all three of these interventions are beggar-thy-neighbor policies: starting at global free trade, each of the described interventions leads to a welfare loss for the trading partner.

We next consider efficient trade policies. To begin, we show that countries can effect lump-sum transfers through tariff changes. This means that efficient tariffs maximize the sum of the two countries' welfare functions. We then confirm that the logic of the entry-externality effect extends to the two-country model of trade; specifically, starting at global free trade, the introduction of a small import or export tariff lowers joint welfare. Hence, starting at global free trade, efficiency is sure to be enhanced in this model when small import or export subsidies are introduced. In a related experiment, we consider a starting point such that, along each direction of trade, the overall trade barrier is zero. This allows for global free trade but includes other policy specifications as well, such as a home import tariff that is exactly offset by a foreign export subsidy. For this family of starting points, we show that the introduction of small tariff changes that induce a small and symmetric increase in the overall trade barrier along each direction of trade is sure to lower joint welfare.

Finally, we consider Nash trade policies. For this analysis, we assume that the joint welfare function is quasi-concave in the overall trade barrier when that barrier takes a symmetric value for each direction of trade. Under this assumption, we show that the symmetric Nash equilibrium is inefficient with an overall barrier that is too high. Thus, starting at the symmetric Nash equilibrium, countries mutually gain by symmetrically exchanging small reductions in import tariffs, export tariffs, or combinations thereof. Under quasi-concavity, we argue further that the overall trade barrier is also too high under global free trade. This reflects the entry-externality effect, under which a trade subsidy improves joint welfare.

This paper builds on our earlier paper, Bagwell and Lee (2018), in which we conduct a similar analysis of trade policies but for the model of Melitz and Ottaviano (2008).[2] In the Melitz-Ottaviano (MO) model, preferences for the differentiated sector are quadratic and, correspondingly, markups are variable. In both papers, we consider a symmetric, two-country, two-sector model in which firm-level productivities (or costs) are drawn from a Pareto distribution. Many trade-policy results hold in common across the two models, as both models feature a selection effect and entry patterns that generate a Metzler paradox. The models differ importantly, however, as regards the characterization of the entry-externality effect. Whereas the externality from additional entry in the CES model is always positive, the sign of the externality in the MO model depends on a simple relationship among parameters. In particular, it is possible in the MO model that too much entry occurs in the market equilibrium of the closed-economy model.

This difference in turn underlies key differences in the policy implications of the two models. Starting at global free trade, if a country introduces a small export subsidy, then in both models the trading partner is harmed; however, the introduction of a small export subsidy is always attractive to the intervening country in the model with CES preferences whereas the appeal of a small export subsidy to the intervening country depends on parameters in the MO model. Starting at global free trade, if the parameters in the MO model are such that the marginal entrant lowers joint welfare, then an export subsidy may shrink the global "pie" to such an extent that a small export subsidy would lower

[2] See also Bagwell and Staiger (2012). They establish some related results for a Cournot oligopoly model with endogenous entry, linear demand and homogeneous firms. An important difference is that they find that free trade is an efficient symmetric policy for that model. They build on Venables (1985).

the welfare of the intervening country. Similarly, while our results for the CES model indicate that a trade subsidy improves joint welfare starting at free-trade benchmarks, a trade subsidy raises joint welfare from such benchmarks in the MO model if and only if model parameters are such that too little entry occurs from a global perspective.

Correspondingly, the models deliver different perspectives on the prohibition of export subsidies in the WTO. For the MO model, and as we discuss in detail in Bagwell and Lee (2018), if countries start at global free trade and model parameters are such that too much entry occurs from a global perspective, then it is possible that the introduction of a small export subsidy could be unilaterally attractive to the intervening country and yet cause a reduction in joint welfare. A prohibition on export subsidies would be effective and efficiency enhancing for this situation. By contrast, for the model with CES preferences considered in this paper, if countries start at global free trade, then the introduction of a small export subsidy is sure to benefit the intervening country and raise joint welfare as well. While a case still might be advanced for restricting export subsidies as a means to limit beggar-thy-neighbor effects, the rationale for a restriction on export subsidies in this setting would not be based on efficiency.

Given that the models have different policy implications, it is important to consider the source of the differences. In Bagwell and Lee (2018), we explore this issue. For a closed-economy setting and starting at the market equilibrium, we show there that additional entry lowers profit conditional on survival for firms in the MO model whereas conditional profit is insensitive to entry in the CES model. An additional channel for the "business-stealing" externality is thus operative in the MO model. This finding offers a partial perspective for why additional entry above the market level can lower welfare in the MO model even though additional entry always raises welfare in the CES model.

Related Work This paper is related to a large literature that studies trade policy in a monopolistic competition model with CES preferences but homogeneous firms. Venables (1987) offers a first model of this kind. He shows that the introduction of a small import tariff can improve welfare in the intervening country, due to the resulting fall in the domestic price index. Helpman and Krugman (1989), Bagwell and Staiger (2015) and Ossa (2011) extend the analysis in various ways. Bagwell and Staiger and also Helpman and Krugman consider a quasi-linear utility function as we do here, whereas Venables and Ossa assume that the utility function takes a Cobb-Douglas structure so that the expenditure shares in the differentiated and outside sectors are fixed.[3]

Campolmi et al (2014) build on Venables' model and show that the free-trade outcome is inefficient, with too few varieties, and establish that a wage subsidy can implement the

[3] For our purposes, an advantage of the quasi-linear specification is that it facilitates a straightforward comparison between our findings here and those that we derive in Bagwell and Lee (2018) for the Meltiz and Ottaviano (2008) model, which also employs a quasi-linear utility function.

first-best outcome. They also remark (see their footnote 13) that, for a model with trade policies only, import and export subsidies can be used in a second-best fashion to improve upon the free-trade allocation. As well, starting at global free trade and when only trade policies are available, they show that the home country gains from the introduction of a small import tariff and from the introduction of a small export subsidy. When only trade policies are available, they also characterize the symmetric Nash equilibrium in import tariffs when export tariffs are fixed at free trade, and they likewise characterize the Nash equilibrium in export tariffs when import tariffs are fixed at free trade. For these settings, they find that import tariffs and export subsidies are used, respectively.

Our paper is related to that of Campolmi et al but differs in key respects. A main difference is that we consider trade policies in a model with heterogeneous firms. In addition, Campolmi et al include domestic policies (a wage subsidy) and thus characterize policy scenarios that we do not consider, but we also characterize trade-policy scenarios that Campolmi et al do not consider. Starting at global free trade, we show that a country can gain with the simultaneous introduction of an import and export tariff. This finding captures a complementary relationship between a country's import and export tariffs. Relatedly, in our characterization of symmetric Nash trade policies, we assume that countries simultaneously select import and export tariffs.[4] For the symmetric Nash trade policies so determined, we also characterize the efficiency properties and associated efficiency-enhancing liberalization paths. Finally, we provide a formal analysis of the entry-externality effect that underlies the inefficient market allocation for the heterogeneous-firms model with CES preferences.

Our work also relates to a small literature that builds on the Melitz (2003) model and considers trade policy under monopolistic competition with CES preferences when firms are heterogeneous. Felbermayr et al (2013) characterize Nash trade policies for the one-sector Melitz model. Demidova and Rodriguez-Clare (2009) consider a small-country version of the model and show that the optimal unilateral export policy is an export tariff. For a two-sector model that includes an outside good, Haaland and Venables (2016) consider a family of small-country models and analyze optimal unilateral trade and domestic taxes. Costinot et al (2016) generalize the one-sector modeling environment in several respects and characterize optimal unilateral tariffs both when tariffs can discriminate across firms and when such discrimination is infeasible. They analyze optimal unilateral tariffs under the assumption that each country has a full set of domestic and trade policy instruments. They also offer a generalization of Haaland and Venables' findings for the

[4] Campolmi et al characterize the symmetric Nash equilibrium when import tariffs and export tariffs are simultaneously determined only when wage subsidies are also determined at the same time. They thus include an instrument (the wage subsidy) with which to target the monopolistic distortion, whereas in our model domestic policies are unavailable and hence each country selects its Nash trade policies with markups in mind.

two-sector model under the small-country restriction. In comparison to these studies, we consider a large-country, two-sector model with a linear outside good, and we assume that governments do not have domestic instruments. We then characterize and compare unilaterally optimal, efficient and Nash symmetric trade policies.

Recent work by Caliendo et al (2017) is also related. They analyze import tariffs in the context of a multi-sector generalization of the Melitz model. Most closely related to the current paper is their analysis of tariffs in a two-sector, two-country model. Our two-sector model has a very different structure but similarly implies that too little entry occurs into the differentiated sector.[5] When the countries start at free trade, Caliendo et al provide conditions under which a country can gain from the unilateral introduction of a small import subsidy; by contrast, we find that a country would always lose from the unilateral introduction of a small import subsidy. We also analyze other unilateral policy interventions and characterize efficient and symmetric Nash tariffs.

Our paper is also related to recent papers that build on the MO model and examine trade policy under monopolistic competition when firms are heterogeneous and preferences are quadratic. As discussed above, Bagwell and Lee (2018) consider a model similar to the one examined here but with the quadratic preference specification. Demidova (2017) characterizes optimal unilateral import tariffs for small and large countries when the outside good is removed from the MO model. A key finding is that the Metzler paradox fails to hold in this setting, due to the associated wage effects of trade policy. Spearot (2014, 2016) builds on the MO model and examines trade policy while allowing for heterogeneous dispersion parameters.

Our analysis of the entry-externality effect is also broadly related to recent work by Dhingra and Morrow (forthcoming) and Nocco et al (2014, forthcoming). Dhingra and Morrow consider a family of one-sector monopolistic competition models with heterogeneous firms, and they show that the market outcome is first best under CES preferences. By contrast, we include a second sector in the form of an outside good, and our analysis of the entry-externality effect relates to a second-best policy (entry subsidies). Our analysis is distinct from Nocco et al, since they consider the efficiency properties of the market outcome in the MO model and examine different second-best policy interventions.

More generally, we can understand our work as offering a tractable "partial-equilibrium" version of the Melitz (2003) model in which to study unilateral, efficient and Nash trade policies. The modeling framework features a quasi-linear utility function into which the outside good enters in a linear and additive fashion. Similar frameworks are commonly used in models with homogeneous firms (see, e.g., Helpman and Krugman, 1989).

[5] Caliendo et al model import tariffs as applying to intermediate goods in the differentiated sector, and their second-sector good is not traded. We do not include intermediate goods and production linkages. Caliendo et al also assume that agents have a Cobb-Douglas utility function and thus have fixed expenditure shares on consumed goods,

Finally, we emphasize that our assumption of a freely traded outside sector serves to eliminate general-equilibrium forces through which trade policies may affect wages. This is an important feature of our modeling framework. Indeed, Demidova and Rodriguez-Clare (2013) argue that the addition of an outside sector to the Melitz (2003) model affects the welfare implications of unilateral changes in iceberg trade costs: when a country experiences a unilateral reduction in the iceberg trade costs for its imports, the country suffers a welfare loss when an outside sector is included whereas for the standard one-sector Melitz model the country experiences a welfare gain.[6] Our modeling framework focuses on import and export tariffs (rather than iceberg trade costs) and seems reasonable for analyzing tariff policies that concern specific sectors, as is often the case in WTO disputes, for example. Sector-level tariffs naturally alter entry patterns and pricing in that sector but are less likely to generate economy-wide wage effects.

The paper is organized as follows. The basic model is developed in Section 2, and the driving forces are featured in Section 3. Unilateral trade policies are considered in Section 4, and efficient and Nash trade policies are examined in Section 5. Concluding comments are provided in Section 6. Omitted proofs are found in the Appendix section.

2 Model

We consider a two-country model of trade between symmetric countries, home (H) and foreign (F). The markets are segmented, and international trade is costly due to trade costs and ad valorem export and import tariffs. The model features free entry, heterogeneous firms, CES preferences and a freely traded outside good.

Consumer Behavior We assume that each country $l \in \{H, F\}$ contains a unit measure of identical consumers. All consumers in country $l \in \{H, F\}$ share the same quasi-linear utility function with CES preferences for the differentiated sector and solve the following problem:

$$U^l \equiv \max_{\left\{q_0^l, \left\{q_i^l\right\}_{i\in\Omega^l}\right\}} q_0^l + \frac{1}{\theta}\left(\int_{i\in\Omega^l} \left(q_i^l\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\theta\cdot\sigma}{\sigma-1}} \tag{1}$$

s.t.

$$q_0^l + \int_{i\in\Omega^l} p_i^l \cdot q_i^l di \leq I^l \equiv w^l + \Pi^l + TR^l \tag{2}$$

where q_0^l, q_i^l and p_i^l represent the consumption of the outside (numeraire) good in country l, the consumption of the differentiated variety $i \in \Omega^l$ in country l, and the price of the

[6] See Demidova (2008) for an analysis of the Melitz model when an outside good is included and the consumer utility function takes a Cobb-Douglas form. She allows for technological asymmetries across countries and examines the implications of a reduction in a symmetric iceberg trade cost.

differentiated variety i in country l. The parameter $\theta \in (0,1)$ indexes the substitution pattern between consumption in the differentiated sector and consumption of the outside good, and the parameter σ refers to an elasticity of substitution between differentiated goods.[7] As shown in (2), consumer's income I^l consists of labor income at the wage w^l, aggregate profit Π^l (which will be zero under free entry), and government transfers of tariff revenue TR^l. We assume that the outside good is consumed in positive quantity, $q_0^l > 0$.[8]

The price index P^l in the differentiated sector can be written as

$$P^l \equiv \left(\int_{i \in \Omega^l} \left(p_i^l\right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \tag{3}$$

Given the price index, we write the demand for the outside good as

$$q_0^l = I^l - \left(P^l\right)^{-\frac{\theta}{1-\theta}} \tag{4}$$

and the demand for differentiated good at variety i as

$$q^l\left(p_i\right) = \left(p_i\right)^{-\sigma} \left(P_i^l\right)^{\frac{\sigma(1-\theta)-1}{1-\theta}} \tag{5}$$

where we assume $\sigma\left(1-\theta\right) > 1$.

Using (1), (2), (3), (4), and (5), we derive the welfare formula for country l as

$$U^l = CS^l + I^l \tag{6}$$

where U^l is indirect utility and consumer surplus CS^l can be written as a function of the price index in country l:

$$CS^l \equiv \left(\frac{1-\theta}{\theta}\right) \left(P^l\right)^{-\frac{\theta}{1-\theta}}. \tag{7}$$

Firm Behavior Labor can be used to produce the outside good under constant returns to scale in a one-to-one manner, where the outside good is sold in a competitive market and is freely traded across countries. We thus treat the outside good as the numeraire and set the wage in each country equal to one: $w^l = 1$.

In the differentiated sector, each variety $i \in \Omega^l$ is produced by a monopolistically competitive firm. To enter the market, a firm pays a fixed entry cost $f_e > 0$ and draws its productivity φ from a common distribution with C.D.F. $G\left(\varphi\right)$. Depending on its

[7] This utility function is also used by Bagwell and Staiger (2016), Helpman and Itskhoki (2010) and Helpman and Krugman (1989).

[8] In the model developed below, this assumption is assured if, for example, trade policies are symmetric and the fixed cost (f_e) for entry into the differentiated sector is sufficiently high.

productivity draw, a firm decides whether to enter market $l \in \{H, F\}$. A firm located in country l with productivity φ solves the following profit maximization problems:

$$\pi_D^l(\varphi) \equiv \max_p p \cdot q^l(p) - \frac{1}{\varphi} q^l(p) - f_D \tag{8}$$

$$\pi_X^l(\varphi) \equiv \max_p \frac{p}{\chi^h} \cdot q^h(p) - \frac{\tau}{\varphi} q^h(p) - f_X \tag{9}$$

where f_D is the fixed cost of production for domestic sales, f_X is the fixed cost of production for foreign sales (exports), $\pi_D^l(\varphi)$ is the profit to an active firm from sales in the domestic market, and $\pi_X^l(\varphi)$ is the profit to an active firm from sales in the foreign market. We assume that $f_X > f_D > 0$.

As indicated in (9), an exporter pays the trade cost $\tau > 0$ and also faces an overall trade barrier χ^h due to trade policies where

$$\chi^h\left(t^h, \tilde{t}^l\right) \equiv \frac{1 + t^h}{1 - \tilde{t}^l},$$

$t^h > -1$ refers to the import tariff levied by country h, and $\tilde{t}^l < 1$ refers to the export tariff levied by country l. Thus, the ad valorem export tariff $\tilde{t}^l$ is levied on the exporting firm with the factory-gate price (i.e., $\frac{p}{1+t^h}$) used for valuation, and similarly the ad valorem import tariff t^h is paid by the importing consumer again with the factory-gate price used for valuation. Under our assumptions, we note that $\chi^h > 0$ and that χ^h is increasing in t^h and $\tilde{t}^l$, where $\chi^h > 1$ if and only if $t^h + \tilde{t}^l > 0$.

By solving (8), we represent the profit-maximizing domestic variables as follows:

$$\begin{aligned}
p_D^l(\varphi) &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi} && (10)\\
q_D^l(\varphi) &= \left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi}\right)^{-\sigma}\left(P^l\right)^{\frac{\sigma(1-\theta)-1}{1-\theta}} && (11)\\
r_D^l(\varphi) &= p_D^l(\varphi)\, q_D^l(\varphi) = \left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi}\right)^{1-\sigma}\left(P^l\right)^{\frac{\sigma(1-\theta)-1}{1-\theta}} && (12)\\
\pi_D^l(\varphi) &= \frac{1}{\sigma} r_D^l(\varphi) - f_D. && (13)
\end{aligned}$$

Thus, $p_D^l(\varphi)$ and $q_D^l(\varphi)$ are the respective profit-maximizing consumption price and quantity produced for the domestic market for an active firm with productivity φ. The corresponding revenue and profit functions are given by $r_D^l(\varphi)$ and $\pi_D^l(\varphi)$, respectively.

Using (9), (10), (11), and (12), we represent profit-maximizing export variables as follows:

$$p_X^l(\varphi) = p_D^h\left(\frac{\varphi}{\tau \cdot \chi^h}\right) \tag{14}$$

$$q_X^l(\varphi) = q_D^h\left(\frac{\varphi}{\tau \cdot \chi^h}\right)$$

$$r_X^l(\varphi) = \frac{1}{\chi^h} p_X^l(\varphi) q_X^l(\varphi) = \frac{1}{\chi^h} r_D^h\left(\frac{\varphi}{\tau \cdot \chi^h}\right) \tag{15}$$

$$\pi_X^l(\varphi) = \frac{1}{\sigma} r_X^l(\varphi) - f_X = \frac{1}{\sigma}\frac{1}{\chi^h} r_D^h\left(\frac{\varphi}{\tau \cdot \chi^h}\right) - f_X. \tag{16}$$

Hence, for a firm active in the export market with productivity level φ, $p_X^l(\varphi)$ and $q_X^l(\varphi)$ are the respective profit-maximizing price and quantity for the export market. The corresponding revenue and profit functions are given by $r_X^l(\varphi)$ and $\pi_X^l(\varphi)$, respectively.

Looking at (10) and (14), we see that for a given variety the price for domestic sales, $p_D^l(\varphi)$, is independent of trade policy whereas the consumption price for exported units is given as $p_X^l(\varphi) = p_D^h\left(\frac{\varphi}{\tau \cdot \chi^h}\right) = \frac{\sigma}{\sigma-1}\frac{\tau \cdot \chi^h}{\varphi}$ and is thus increasing in the overall trade barrier due to trade policies, χ^h. We note further that the factory-gate price for exported units is given as $\frac{p_X^l(\varphi)}{1+t^h} = \frac{\sigma}{\sigma-1}\frac{\tau}{\varphi}\frac{1}{1-\tilde{t}^l}$. Thus, for the CES model considered here, the factory-gate price for exports from country l is unaffected by the import tariff in country h but is increasing in the export tariff in country l.

A firm enters the domestic (foreign) market if and only if $\varphi > \varphi_D^{l*}$ ($\varphi > \varphi_X^{l*}$) where these productivity cutoff levels are determined by *Zero Cutoff Profit (ZCP)* conditions

$$\pi_D^l\left(\varphi_D^{l*}\right) = 0 \text{ and } \pi_X^l\left(\varphi_X^{l*}\right) = 0. \tag{17}$$

By (12), (13), (15), (16), and (17), we can link φ_X^{h*} to φ_D^{l*} as

$$\varphi_X^{h*} = A^l \cdot \varphi_D^{l*} \text{ where } A^l \equiv \tau \cdot \left(\chi^l\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{f_X}{f_D}\right)^{\frac{1}{\sigma-1}}. \tag{18}$$

Price Index We now show that the price index in country l can be written in terms of the productivity cutoff level, φ_D^{l*}. To do this, we derive and relate two different expressions for the price index.

The first relationship utilizes the productivity cut-off levels determined by (17) and (18). Using these, we write the "average" productivity of domestic sellers $\tilde{\varphi}_D^l$ and the "average" productivity of exporters $\tilde{\varphi}_X^l$ in country l as

$$\begin{aligned}\tilde{\varphi}_D^l &\equiv \tilde{\varphi}\left(\varphi_D^{l*}\right) \\ \tilde{\varphi}_X^l &\equiv \tilde{\varphi}\left(\varphi_X^{l*}\right),\end{aligned} \tag{19}$$

where $\tilde{\varphi}(\varphi^*)$ is the "average" productivity of operating firms given the productivity cutoff

φ^*:

$$\tilde{\varphi}(\varphi^*) \equiv \left(\frac{\int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi)}{1 - G(\varphi^*)} \right)^{\frac{1}{\sigma-1}}. \tag{20}$$

Let the total mass of varieties available in country l be denoted as N^l, where N^l is comprised of the mass of domestic sellers, N_D^l, and the mass of exporters from country h, N_X^h:

$$N^l = N_D^l + N_X^h.$$

Using $\tilde{\varphi}_D^l$ and $\tilde{\varphi}_X^h$, we write the "average" productivity of all firms competing in country l (where the productivity of exporters is adjusted by trade and tariff costs, $\tau \cdot \chi^l$) as

$$\tilde{\varphi}_T^l \equiv \left(\frac{1}{N^l} \left[N_D^l \cdot \left(\tilde{\varphi}_D^l\right)^{\sigma-1} + N_X^h \cdot \left(\frac{\tilde{\varphi}_X^h}{\tau \cdot \chi^l} \right)^{\sigma-1} \right] \right)^{\frac{1}{\sigma-1}}. \tag{21}$$

With these definitions in place, we now note that the price index as defined in (3) may be represented as

$$P^l = \left(N_D^l \cdot \frac{\int_{\varphi_D^{l*}}^{\infty} (p_D^l(\varphi))^{1-\sigma} dG(\varphi)}{1 - G(\varphi_D^{l*})} + N_X^h \cdot \frac{\int_{\varphi_X^{h*}}^{\infty} (p_X^h(\varphi))^{1-\sigma} dG(\varphi)}{1 - G(\varphi_X^{h*})} \right)^{\frac{1}{1-\sigma}} \tag{22}$$

Using (10), (14), (19), (20) and (21), we may now simplify the price index further as

$$P^l = \left(N^l\right)^{\frac{1}{1-\sigma}} \cdot p_D^l\left(\tilde{\varphi}_T^l\right). \tag{23}$$

By rearranging (23) and using (10), we may write N^l in terms of $\tilde{\varphi}_T^l$ and P^l :

$$N^l = \left(\tilde{\varphi}_T^l\right)^{1-\sigma} \left(\frac{\sigma}{\sigma - 1} \frac{1}{P^l} \right)^{\sigma-1}. \tag{24}$$

The second expression for the price index builds from out knowledge of aggregate expenditure in the differentiated sector. We know from (2) and (4) that the aggregate expenditure by consumers in country l in the differentiated sector can be written as $\left(P^l\right)^{\frac{-\theta}{1-\theta}}$. The aggregate expenditure by consumers in country l in the differentiated sector can also be represented in terms of revenue functions as

$$N_D^l \cdot \frac{\int_{\varphi_D^{l*}}^{\infty} r_D^l(\varphi)\, dG(\varphi)}{1 - G(\varphi_D^{l*})} + N_X^h \cdot \frac{\int_{\varphi_X^{h*}}^{\infty} r_X^h(\varphi)\, \chi^l dG(\varphi)}{1 - G(\varphi_X^{h*})} = N^l \cdot r_D^l\left(\tilde{\varphi}_T^l\right),$$

where $p_X^h(\varphi)\, q_X^h(\varphi) = r_X^h(\varphi)\, \chi^l$ follows from (15) and where the equality can be established by using (12), (15), (19), (20) and (21). Thus, we obtain

$$\left(P^{l}\right)^{\frac{-\theta}{1-\theta}}=N^{l}\cdot r_{D}^{l}\left(\tilde{\varphi}_{T}^{l}\right). \tag{25}$$

Next, we use (12) and observe that the revenue of a firm with productivity φ in country l can be written as

$$r_{D}^{l}\left(\varphi\right)=\left(\frac{\varphi}{\varphi_{D}^{l*}}\right)^{\sigma-1}r_{D}^{l}\left(\varphi_{D}^{l*}\right)=\left(\frac{\varphi}{\varphi_{D}^{l*}}\right)^{\sigma-1}\sigma\cdot f_{D} \tag{26}$$

where the second equality holds by ZCP as captured by (17), which yields $r_{D}^{l}\left(\varphi_{D}^{l*}\right)=\sigma\cdot f_{D}$. Using (25) and (26), we can write N^{l} as

$$N^{l}=\frac{\left(P^{l}\right)^{\frac{-\theta}{1-\theta}}}{\left(\frac{\tilde{\varphi}_{T}^{l}}{\varphi_{D}^{l*}}\right)^{\sigma-1}\sigma\cdot f_{D}}. \tag{27}$$

By equating (24) and (27), we may now pin down price index P^{l} as a function of domestic productivity cutoff:

$$P^{l}=\left(\sigma\cdot f_{D}\right)^{\frac{1-\theta}{\sigma(1-\theta)-1}}\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi_{D}^{l*}}\right)^{\frac{(1-\theta)(\sigma-1)}{\sigma(1-\theta)-1}}. \tag{28}$$

From (28), we see that tougher selection lowers the price index: $\frac{dP^{l}}{d\varphi_{D}^{l*}}<0$.

Free Entry Condition Under free entry, the expected profit from obtaining a productivity draw must be zero. Thus, for $l\in\{H,F\}$, the following condition must hold:

$$\int_{\varphi_{D}^{l*}}^{\infty}\pi_{D}^{l}\left(\varphi\right)dG(\varphi)+\int_{\varphi_{X}^{l*}}^{\infty}\pi_{X}^{l}\left(\varphi\right)dG(\varphi)=f_{e}. \tag{29}$$

Our next task is to express (29) in terms of the productivity cutoff levels, $\left\{\varphi_{D}^{l*}\right\}_{l\in\{H,F\}}$.

To this end, we may use (13) and (26) to derive that

$$\pi_{D}^{l}\left(\varphi\right)=f_{D}\cdot[(\frac{\varphi}{\varphi_{D}^{l*}})^{\sigma-1}-1]. \tag{30}$$

Using (30) along with (19) and (20), we can show that

$$\int_{\varphi_{D}^{l*}}^{\infty}\pi_{D}^{l}\left(\varphi\right)dG(\varphi)=\pi_{D}^{l}\left(\tilde{\varphi}_{D}^{l}\right)\cdot(1-G(\varphi_{D}^{l*})).$$

Similarly, using (16), (18) and (26), we find that

$$\pi_X^l(\varphi) = f_X \cdot [(\frac{\varphi}{\varphi_X^{l*}})^{\sigma-1} - 1]. \tag{31}$$

Using (31) along with (19) and (20), we find that

$$\int_{\varphi_X^{l*}}^{\infty} \pi_X^l(\varphi)\, dG(\varphi) = \pi_X^l\left(\tilde{\varphi}_X^l\right) \cdot (1 - G(\varphi_X^{l*})).$$

The condition for zero expected profit in (29) can thus be rewritten as

$$\pi_D^l\left(\tilde{\varphi}_D^l\right) \cdot (1 - G(\varphi_D^{l*})) + \pi_X^l\left(\tilde{\varphi}_X^l\right) \cdot (1 - G(\varphi_X^{l*})) = f_e.$$

To put this condition in a more compact form, we refer to (30) and write the "average" profit from domestic operation as

$$\pi_D^l\left(\tilde{\varphi}_D^l\right) = f_D \cdot K\left(\varphi_D^{l*}\right)$$

where function K is defined as

$$K(\varphi^*) \equiv \left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1. \tag{32}$$

Similarly, we refer to (31) and write the "average" profit from exporting as

$$\pi_X^l\left(\tilde{\varphi}_X^l\right) = f_X \cdot K\left(\varphi_X^{l*}\right).$$

Pulling these findings together, we may now express the *free entry condition* as

$$\left(1 - G\left(\varphi_D^{l*}\right)\right) \cdot f_D \cdot K\left(\varphi_D^{l*}\right) + \left(1 - G\left(\varphi_X^{l*}\right)\right) \cdot f_X \cdot K\left(\varphi_X^{l*}\right) = f_e \tag{33}$$

for $l \in \{H, F\}$. Since we know $\varphi_X^{l*} = A^h \cdot \varphi_D^{h*}$ from (18), (33) is a two-equation system that pins down $\left\{\varphi_D^{l*}\right\}_{l\in\{H,F\}}$.

Pareto Distribution Following Chaney (2008), we assume that φ follows the Pareto distribution with shape parameter k so that $G(\varphi) = 1 - \varphi^{-k}$ for $\varphi \in [1, \infty)$. Using this distributional assumption, we can pin down the productivity cutoffs $\left\{\varphi_D^{l*}\right\}_{l\in\{H,F\}}$ in closed form in terms of parameters.

We first rewrite (20) and (32) as

$$\tilde{\varphi}(\varphi^*) = (\frac{k}{1+k-\sigma})^{\frac{1}{\sigma-1}} \cdot \varphi^* \tag{34}$$

$$K(\varphi^*) = \frac{\sigma - 1}{1 + k - \sigma} \tag{35}$$

where we keep Chaney's assumption that $1 + k - \sigma > 0$. Then, using (18) and (35), we may rewrite (33) as

$$f_D \cdot \left(\varphi_D^{l*}\right)^{-k} + f_X \cdot \left(A^h \cdot \varphi_D^{h*}\right)^{-k} = \phi \text{ for } l \in \{H, F\} \tag{36}$$

where $\phi \equiv (1 + k - \sigma) f_e / (\sigma - 1) > 0$. The solutions to (36) can be written as

$$\varphi_D^{h*} = \left(\frac{\left(1 - \frac{f_X}{f_D}\left(A^l\right)^{-k}\right)\phi}{f_D\left(1 - \left(\frac{f_X}{f_D}\right)^2 \cdot \left(A^l \cdot A^h\right)^{-k}\right)} \right)^{-1/k} \quad \text{for } h \in \{H, F\}. \tag{37}$$

In order to guarantee $\varphi_D^{h*} > 0$, we maintain the assumption that, at the tariffs of interest,

$$1 - \frac{f_X}{f_D}\left(A^l\right)^{-k} > 0 \text{ for } l \in \{H, F\} \tag{38}$$

which implies

$$1 - \left(\frac{f_X}{f_D}\right)^2 \cdot \left(A^l \cdot A^h\right)^{-k} > 0. \tag{39}$$

Given our assumptions that $1 + k - \sigma > 0$, $\theta \in (0, 1)$, $\sigma > 1/(1 - \theta)$ and (38), we may refer to the implied (39) and confirm from (37) that $\varphi_D^{h*} > 0$ is indeed guaranteed.[9,10]

Using (18), we note that (38) holds under global free trade (implying $\chi^l = \chi^h = 1$) if and only if

$$\tau > \left(\frac{f_D}{f_X}\right)^{\frac{1+k-\sigma}{k(\sigma-1)}}. \tag{40}$$

We maintain this assumption throughout. Given (40), we can be sure that $\varphi_D^{h*} > 0$ when tariffs are close enough to zero.

Tariff Revenue As (2) shows, consumer income is composed of a unit of labor income, aggregate profit, and tariff revenue. We have already discussed that labor income is given by $w^l = 1$ and that the free entry condition implies zero aggregate profit $\Pi^l = 0$. The remaining income source to consider is tariff revenue.

We define $IMP^l(EXP^l)$ as the value of country l's imports (exports) prior to the

[9] Using (18) and (38), we can also confirm that $A^h > 1$ follows from our assumption that $f_X > f_D$, so that $\varphi_X^{l*} > \varphi_D^{h*}$ must hold at the tariffs of interest as well.

[10] We note that (38) has a counterpart in Demidova's (2008) analysis. She provides a necessary condition to her "Assumption 2" on p. 1450, which guarantees the existence of a unique solution to the free entry condition. Her necessary condition is equivalent to (38) under our setup.

imposition of the import tariff:

$$IMP^l = \frac{N_E^h}{1+t^l} \int_{\varphi_X^{h*}}^{\infty} p_X^h(\varphi)\, q_X^h(\varphi)\, dG(\varphi) = \frac{N_E^h}{1-\tilde{t}^h} \left(\varphi_X^{h*}\right)^{-k} \frac{k \cdot \sigma \cdot f_X}{1+k-\sigma} \tag{41}$$

$$EXP^l = \frac{N_E^l}{1+t^h} \int_{\varphi_X^{l*}}^{\infty} p_X^l(\varphi)\, q_X^l(\varphi)\, dG(\varphi) = \frac{N_E^l}{1-\tilde{t}^l} \left(\varphi_X^{l*}\right)^{-k} \frac{k \cdot \sigma \cdot f_X}{1+k-\sigma}. \tag{42}$$

The second equalities in (41) and (42) are driven by (15), (18), (20), (26) and (34).

Hence, in country l, tariff revenue is given as

$$TR^l = t^l \cdot IMP^l + \tilde{t}^l \cdot EXP^l. \tag{43}$$

Number of Entrants To recover the number of entrants in this model, we begin by relating the number of entrants in the two countries to the number of domestic sellers in country l and exporters from country h:

$$N_D^l = (1 - G(\varphi_D^{l*})) \cdot N_E^l \tag{44}$$

$$N_X^h = (1 - G(\varphi_X^{h*})) \cdot N_E^h. \tag{45}$$

Next, we combine (44) and (45) with (22) to get for $l \in \{H, F\}$ that

$$P^l = \left(N_E^l \cdot \int_{\varphi_D^{l*}}^{\infty} (p_D^l(\varphi))^{1-\sigma} dG(\varphi) + N_E^h \cdot \int_{\varphi_X^{h*}}^{\infty} (p_X^h(\varphi))^{1-\sigma} dG(\varphi) \right)^{\frac{1}{1-\sigma}}, \tag{46}$$

a system of two equations.

To solve the system, we use (10), (12), (14) and (15) in order to rewrite (46) as

$$\left(P^l\right)^{-\frac{\theta}{1-\theta}} = N_E^h \cdot T_1^l + N_E^l \cdot T_2^l \text{ for } l \in \{H, F\} \tag{47}$$

where

$$\begin{aligned} T_1^l &= \chi^l \int_{\varphi_X^{h*}}^{\infty} r_X^h(\varphi)\, dG(\varphi) = \chi^l \left(\varphi_X^{h*}\right)^{-k} \frac{k \cdot \sigma \cdot f_X}{1+k-\sigma} \\ T_2^l &= \int_{\varphi_D^{l*}}^{\infty} r_D^l(\varphi)\, dG(\varphi) = \left(\varphi_D^{l*}\right)^{-k} \frac{k \cdot \sigma \cdot f_D}{1+k-\sigma}. \end{aligned} \tag{48}$$

The second equality in the expression for T_1^l follows after using (15) and (41); and the second equality in the expression for T_2^l follows after using (20), (26) and (34).

For $l \in \{H, F\}$, we note the following dependencies: using (18), T_1^l and T_2^l are functions of φ_D^{l*}; using (28), P^l is a function of φ_D^{l*}; and using (18), φ_D^{l*} is determined given tariffs by (37). Thus, we may solve the two-by-two equation system in (47) to pin down the number of entrants in each country as

$$N_E^l = \frac{T_2^h \left(P^l\right)^{-\frac{\theta}{1-\theta}} - T_1^l \left(P^h\right)^{-\frac{\theta}{1-\theta}}}{T_2^l \cdot T_2^h - T_1^l \cdot T_1^h} \text{ for } l \in \{H, F\}, \tag{49}$$

where N_E^l depends on tariffs in (49) only through φ_D^{l*} and φ_D^{h*}.

We now confirm that $N_E^l > 0$ when $\chi^l = \chi^h = 1$, where we note that $\chi^l = \chi^h = 1$ includes the possibility of *global free trade* (i.e., $t^l = t^h = \tilde{t}^l = \tilde{t}^h = 0$). To this end, we use (48) and find that

$$\{T_2^l \cdot T_2^h - T_1^l \cdot T_1^h\}|_{\chi^l=\chi^h=1} = \{\left(\varphi_D^{l*} \cdot \varphi_D^{h*}\right)^{-k} (\frac{k \cdot \sigma \cdot f_D}{1+k-\sigma})^2[1-(A^l A^h)^{-k}(\frac{f_X}{f_D})^2]\}|_{\chi^l=\chi^h=1} > 0,$$

where the inequality follows from (40) which implies $\varphi_D^{l*} > 0$ for $l \in \{H, F\}$. Next, we note from (18), (28) and (37) that $P^l = P^h$ when $\chi^l = \chi^h$. Building from this observation and using (48), we find that

$$\begin{aligned}
\{T_2^h \left(P^l\right)^{-\frac{\theta}{1-\theta}} - T_1^l \left(P^h\right)^{-\frac{\theta}{1-\theta}}\}|_{\chi^l=\chi^h=1} &= \{\left(P^l\right)^{-\frac{\theta}{1-\theta}} (T_2^h - T_1^l)\}|_{\chi^l=\chi^h=1} \\
&= \{\left(P^l\right)^{-\frac{\theta}{1-\theta}} \left(\varphi_D^{l*}\right)^{-k} (\frac{k \cdot \sigma \cdot f_D}{1+k-\sigma})[1-(A^l)^{-k}\frac{f_X}{f_D}]\}|_{\chi^l=\chi^h=1} \\
&> 0,
\end{aligned}$$

where the inequality follows from (40) which implies $\varphi_D^{l*} > 0$ for $l \in \{H, F\}$. We conclude that $N_E^l > 0$ when $\chi^l = \chi^h = 1$; hence, under our assumptions, a positive number of entrants in each country is assured when tariffs are sufficiently close to global free trade.

More generally, in the analysis below, we maintain the assumption that the number of entrants is positive in each country (i.e., $N_E^l > 0$ for $l \in \{H, F\}$) at the tariffs of interest. We can confirm that this assumption is assured by (38) when tariffs are such that $\chi^l = \chi^h$.[11] In subsequent sections, we give particular consideration to trade policies that constitute global free trade. As indicated in the previous paragraph, our maintained assumption for this case can be captured analytically with restrictions on model parameters.

Welfare To complete our description of the model, we return to the representation of welfare in country l. Referring to (2), (6), (7), $\Pi^l = 0$ as captured by (33), and the representation of tariff revenue that comes from (41)-(43), and recalling that $w^l = 1$, we

[11] Given $\chi^l = \chi^h$, we know that $A^l = A^h$, $\varphi_D^{l*} = \varphi_D^{h*}$ and $P^l = P^h$. Substituting (48) into (49) and using these symmetric relationships, we find that $N_E^l = N_E^h > 0$ if (38) holds so that $\varphi_D^{l*} > 0$ and thus $P^l > 0$ by (28).

represent the welfare function for country l as

$$U^l = 1 + t^l \cdot IMP^l + \tilde{t}^l \cdot EXP^l + \left(\frac{1-\theta}{\theta}\right)\left(P^l\right)^{-\frac{\theta}{1-\theta}}, \tag{50}$$

where the last term corresponds to consumer surplus, CS^l, and the other terms combine to form income in country l. We recall from (7) and (28) that CS^l depends on tariffs only through the determination of φ_D^{l*}, and we observe from (41)-(43) that tariffs affect tariff revenue both directly and through the induced long-run impact on trade values, IMP^l and EXP^l, where IMP^l and EXP^l ultimately depend on tariffs through the determination of φ_D^{l*} and φ_D^{h*}.

3 Driving Forces

In this section, we conduct some comparative-statics exercises and identify the driving forces of the model. These forces provide an intuitive foundation for our trade-policy analysis in Sections 4 and 5.

3.1 Selection Effect

We show here that a higher home import tariff (or a higher foreign export tariff) raises the cut-off productivity level for domestic sales in the home market and lowers the cut-off productivity level for domestic sales in the foreign market.

Formally, we establish the following proposition:

Proposition 1 *(selection effect) Tariffs affect selection for domestic sales,*

$$\frac{\partial \varphi_D^{l*}}{\partial t^h}, \frac{\partial \varphi_D^{l*}}{\partial \tilde{t}^l} < 0 < \frac{\partial \varphi_D^{l*}}{\partial \tilde{t}^h}, \frac{\partial \varphi_D^{l*}}{\partial t^l}, \tag{51}$$

and likewise for export sales,

$$\frac{\partial \varphi_X^{l*}}{\partial \tilde{t}^h}, \frac{\partial \varphi_X^{l*}}{\partial t^l} < 0 < \frac{\partial \varphi_X^{l*}}{\partial t^h}, \frac{\partial \varphi_X^{l*}}{\partial \tilde{t}^l}. \tag{52}$$

Proof. Omitted proofs are in the Appendix. ■

As confirmed in the Appendix, this proposition follows easily from (18) and (37), given (38) and the implied (39).[12]

[12] We see from Proposition 1 that, along a given channel of trade, higher home import and foreign export tariffs push the cut-off productivity levels in the same direction. The size of the change may

3.2 Metzler Paradox

Using (28) and (51), we may now directly conclude that the Metzler paradox holds in this model. Formally, we capture the implications of tariffs for price indices with the following proposition:

Proposition 2 *(Metzler paradox) For countries l and h with $l,\ h \in \{H, F\}$ and $l \neq h$, an increase in country l's import tariff or in country h's export tariff results in a decrease in the price index in country l and increase in the price index in country h:*

$$\frac{dP^l}{d\tilde{t}^h}, \frac{dP^l}{dt^l} < 0 < \frac{dP^h}{dt^l}, \frac{dP^h}{d\tilde{t}^h}. \tag{53}$$

Using (7), we note that a lower value for P^l implies a higher level of consumer surplus. The key point is that a higher home import tariff (or a higher foreign export tariff) raises the cut-off productivity level for sales in the domestic market, which in turn generates a lower price index and thus a higher level of consumer surplus in the home country.[13,14]

3.3 Entry-externality Effect

In this subsection, we characterize the entry-externality effect by considering a closed-economy model with a single country. This analysis is of direct interest and also offers an intuitive foundation for understanding our trade-policy findings in the two-country model.

3.3.1 The closed-economy model

We now consider a closed-economy setting where the social planner chooses the number of entrants, N_E. Let P refer to the price index of the closed economy as defined in

vary across trade policies, however. The asymmetry arises since, for a given consumption (i.e., delivered) price $p_X^h(\varphi)$ of a country-h export good for consumers in country l, a higher import tariff t^l lowers the associated factory-gate price, $p_X^h(\varphi)/(1+t^l)$, but a higher export tariff $\tilde{t}^h$ leaves the associated factory-gate price unaffected. For each of the following three inequalities, $\frac{\partial \chi^l}{\partial t^l} < \frac{\partial \chi^l}{\partial \tilde{t}^h}$, $\frac{\partial \varphi_D^{l*}}{\partial t^l} < \frac{\partial \varphi_D^{l*}}{\partial \tilde{t}^h}$ and $\frac{\partial \varphi_D^{h*}}{\partial \tilde{t}^h} < \frac{\partial \varphi_D^{h*}}{\partial t^l}$, we can show as well that the inequality holds if and only if $t^l + \tilde{t}^h > 0$. See also Bagwell and Lee (2018) for related findings in the MO model.

[13] As in footnote 12, we can also report findings about relative magnitudes. For each of the following two inequalities, $\frac{dP^l}{d\tilde{t}^h} < \frac{dP^l}{dt^l}$ and $\frac{dP^h}{dt^l} < \frac{dP^h}{d\tilde{t}^h}$, we can show that the inequality holds if and only if $t^l + \tilde{t}^h > 0$. See also Bagwell and Lee (2018).

[14] If tariffs are symmetric in that $t^l = t^h$ and $\tilde{t}^l = \tilde{t}^h$ and thus so that $\chi^l = \chi^h$, and if (38) and (40) hold, then we can further show that N_E^l (N_E^h) rises (falls) when t^l rises, and that N_E^l (N_E^h) falls (rises) when $\tilde{t}^l$ rises. From this perspective, we may understand that a higher import tariff by country l, for example, leads to additional entry in country l and less entry in country h, resulting in a higher productivity cut-off in country l and a lower cut-off productivity in country h, so that the price index falls in country l and rises in country h.

(3). Under the given price index P (instead of P^l), firms' decisions follow from (10), (11), (12) and (13). The productivity cutoff to serve the closed market is denoted as φ^* and is determined by the ZCP condition for the closed economy, $\pi_D(\varphi^*) = 0$, where $\pi_D(\varphi) = \pi^l_D(\varphi)|_{P^l=P}$.

The expected profit $\overline{\pi}$ for an entrant can be written as a function of the productivity cutoff φ^* :

$$\overline{\pi} \equiv \int_{\varphi^*}^{\infty} \pi_D(\varphi) dG(\varphi) = (\varphi^*)^{-k} \frac{(\sigma-1) f_D}{1+k-\sigma}, \tag{54}$$

where the equality uses (13), (20), the ZCP condition as captured in (26), (32) and (35).

In analogy with our derivation of (23), we can write the price index as a function of N_E and φ^*. To this end, we note that the price index (3) can be represented in the closed-economy setting as

$$P = \left(N \cdot \frac{\int_{\varphi^*}^{\infty} (p_D(\varphi))^{1-\sigma} dG(\varphi)}{1-G(\varphi^*)} \right)^{\frac{1}{1-\sigma}} = \left(N_E \cdot \int_{\varphi^*}^{\infty} (p_D(\varphi))^{1-\sigma} dG(\varphi) \right)^{\frac{1}{1-\sigma}},$$

where $p_D(\varphi) = \frac{\sigma}{\sigma-1}\frac{1}{\varphi}$ follows from (10) and where the number of entrants, N_E, and surviving varieties, N, are related as $N = N_E(1-G(\varphi^*))$. Substituting $p_D(\varphi) = \frac{\sigma}{\sigma-1}\frac{1}{\varphi}$, we now obtain

$$\begin{aligned} P &= (N_E)^{\frac{1}{1-\sigma}} (\frac{\sigma}{\sigma-1}) \left(\int_{\varphi^*}^{\infty} \varphi^{\sigma-1} dG(\varphi) \right)^{\frac{1}{1-\sigma}} \\ &= (N_E)^{\frac{1}{1-\sigma}} (\frac{\sigma}{\sigma-1})(\frac{1+k-\sigma}{k})^{\frac{1}{\sigma-1}} (\varphi^*)^{\frac{1+k-\sigma}{\sigma-1}} \end{aligned} \tag{55}$$

where (55) follows after using (20) and again imposing the Pareto distribution.

In line with our derivation of (25), we set aggregate revenue equal to aggregate expenditure in the differentiated sector. We thus have that

$$(P)^{\frac{-\theta}{1-\theta}} = N_E \cdot \int_{\varphi^*}^{\infty} r_D(\varphi) dG(\varphi) \tag{56}$$

where $r_D(\varphi) = r^l_D(\varphi)|_{P^l=P}$ so that $\pi_D(\varphi) = \frac{1}{\sigma} r_D(\varphi) - f_D$ just as in (12) and (13). We now define and characterize the expected revenue per entrant, $\overline{r}$, as follows:

$$\overline{r} \equiv \int_{\varphi^*}^{\infty} r_D(\varphi) dG(\varphi) = (\varphi^*)^{-k} (\frac{k}{1+k-\sigma}) \sigma \cdot f_D, \tag{57}$$

where the equality uses (12), (20), the ZCP condition as captured in (26) and the imposition of the Pareto distribution. Using (57), we refer back to (56) to obtain

$$(P)^{\frac{-\theta}{1-\theta}} = N_E \cdot (\varphi^*)^{-k} (\frac{k}{1+k-\sigma}) \sigma \cdot f_D \tag{58}$$

as a second expression for the price index in terms of N_E and φ^*, where this expression now uses the ZCP condition.

Using (55) and (58), we may now obtain a one-to-one relation between P and φ^* :

$$P = (\sigma \cdot f_D)^{\frac{1-\theta}{\sigma(1-\theta)-1}} \left(\frac{\sigma}{\sigma - 1} \frac{1}{\varphi^*} \right)^{\frac{(1-\theta)(\sigma-1)}{\sigma(1-\theta)-1}} . \tag{59}$$

We note from (59) that the price index falls when the cut-off productivity level rises: $\frac{dP}{d\varphi^*} < 0$.

By equating (55) and (59), we can now relate φ^* to N_E :

$$\Upsilon \cdot (N_E)^{\frac{1}{\sigma-1}} = (\varphi^*)^{\frac{k+1-\sigma}{\sigma-1} + \frac{(1-\theta)(\sigma-1)}{\sigma(1-\theta)-1}} \tag{60}$$

where

$$\Upsilon = (\sigma \cdot f_D)^{\frac{1-\theta}{\sigma(1-\theta)-1}} \left(\frac{\sigma}{\sigma - 1} \right)^{\frac{(1-\theta)(\sigma-1)}{\sigma(1-\theta)-1}} \left(\frac{\sigma}{\sigma - 1} \right)^{-1} \left(\frac{k}{1 + k - \sigma} \right)^{\frac{1}{\sigma-1}} > 0.$$

For $N_E > 0$, we see from (60) that selection becomes tougher when we have more entrants: $\frac{d\varphi^*}{dN_E} > 0$. Correspondingly, we see from (59) and (60) that the price index then decreases with additional entry: $\frac{dP}{dN_E} < 0$.

We note that (54)-(60) as derived above use the ZCP condition but do not rely on the free-entry condition. Thus, we are free to use those conditions while selecting the level of entry, N_E.[15] Formally, a choice of N_E implies a value for φ^* via (60) and thereby a value for P via (59), a value for $\bar{\pi}$ via (54), and a value for N via $N = N_E(1 - G(\varphi^*))$.

3.3.2 The Entry-Externality Effect

We now consider the problem of social planner who selects N_E with the objective

$$\max_{N_E} CS\,(P) + N_E\,(\bar{\pi} - f_e)\,,$$

where as in (7) we capture consumer surplus as

$$CS\,(P) \equiv \left(\frac{1 - \theta}{\theta} \right) (P)^{-\frac{\theta}{1-\theta}} . \tag{61}$$

[15] A change in the number of entrants can be achieved in a decentralized setting by using lump-sum transfers to subsidize or tax the fixed cost of entry. For example, to implement an increase in N_E relative to the market equilibrium level of entry, the social planner could provide a lump-sum entry subsidy $T_E < 0$. Entry would then occur until $\bar{\pi} - f_e - T_E = 0$. The subsidy would be financed via a lump-sum tax on consumers in the amount of $N_E(\bar{\pi} - f_e)$.

The first-order condition for the socially optimal level of entry, N_E^*, takes the following form:

$$\frac{dCS}{dN_E} + N_E \frac{d\bar{\pi}}{dN_E} + \bar{\pi} - f_e = 0.$$

By contrast, the market determines the entry level to satisfy the free-entry condition, $\bar{\pi} = f_e$. We thus define the externalities that the market does not consider as follows:

$$EXT \equiv \frac{dCS}{dN_E} + N_E \frac{d\bar{\pi}}{dN_E} \tag{62}$$

where for $N_E > 0$ we have that $\frac{dCS}{dN_E} > 0$ follows from (61) and $\frac{dP}{dN_E} < 0$ and captures the consumer surplus gain from additional entry while $\frac{d\bar{\pi}}{dN_E} < 0$ follows from (54) and $\frac{d\varphi^*}{dN_E} > 0$ and captures the business-stealing effect that is associated with additional entry.

We establish next a sense in which the consumer surplus gain always dominates the business-stealing effect. To state our result, we define N_E^m as the level of entry determined by the free-entry condition $\bar{\pi} = f_e$. We assume that $N_E^m > 0$. Our result is as follows:

Proposition 3 *(Entry-externality effect) Starting at the market equilibrium, additional entry always generates a positive externality:*

$$EXT > 0 \text{ if } N_E = N_E^m.$$

Thus, the sign of EXT is always positive in this model. This finding implies that, starting at the market solution, additional entry financed by a lump-sum transfer would raise welfare in the model studied here; in other words, the market supplies insufficient entry in the CES model under consideration here.

As discussed in the Introduction, our finding of insufficient entry in the CES model is of particular interest when contrasted with our finding in Bagwell and Lee (2018) for the MO model. We find for the MO model that, depending on model parameters, the market may provide too little entry or too much entry. We refer readers to Bagwell and Lee (2018) for additional discussion of the distinct entry-externality effects in these two models. We argue there that a key - and special - feature of the CES model is that a firm's profit conditional on survival is independent of N_E.[16]

[16] For the CES model, the conditional expected profit is given by $\bar{\pi}_c = (\sigma - 1) f_D / (1 + k - \sigma)$. We note that this value is independent of φ^* and thus N_E. The derivation of $\bar{\pi}_c$ follows from (54), given that $1 - G(\varphi^*) = (\varphi^*)^{-k}$ for the Pareto distribution.

4 Unilateral Trade Policies

Returning to the two-country model and using the driving forces above, we now examine a government's unilateral incentive to deviate from global free trade.

4.1 Introduction of Small Import Tariff

If countries start at global free trade, then the introduction of a small import tariff by country l raises the welfare of country l and lowers the welfare of country h. The following proposition states this finding in formal terms:

Proposition 4 *(small import tariff) For countries l and h with l, $h \in \{H, F\}$ and $l \neq h$, if both countries initially adopt a policy of free trade, then the introduction of a small import tariff by country l generates a welfare gain for country l and a welfare loss for country h:*

$$\frac{dU^l}{dt^l} \mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} = IMP^l + \frac{dCS^l}{dP^l}\frac{dP^l}{dt^l} \mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} > 0 \tag{63}$$

$$\frac{dU^h}{dt^l} \mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} = \frac{dCS^h}{dP^h}\frac{dP^h}{dt^l} \mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} < 0. \tag{64}$$

We recall that $IMP^l > 0$ at global free trade and note that (63) and (64) follow directly from (7), (50) and the Metzler paradox as captured in (53). Intuitively, country l gains from the imposition of a small import tariff due to the gain in tariff revenue and the higher consumer surplus that is associated with a lower price index in country l. Similarly, country h loses when country l imposes a small import tariff, due to the lower consumer surplus that is a associated with a higher price index in country h.

4.2 Introduction of Small Export Subsidy

The introduction of small export subsidy by country l, starting at global free trade, raises the price index in country h by the Metzler paradox as captured in (53). Hence, starting at global free trade, the introduction of a small export subsidy by country l lowers welfare in country h due to the associated loss in country-h consumer surplus:

$$\frac{dU^h}{d\tilde{t}^l} \mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} = \frac{dCS^h}{dP^h}\frac{dP^h}{d\tilde{t}^l} \mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} > 0, \tag{65}$$

where we use (7), (50) and the Metzler paradox as captured in (53). Since the Metzler paradox is the driving force behind (63), (64) and (65), the findings captured by these expressions are the same as in those that we provide in Bagwell and Lee (2018) for the MO model.

For the subsidizing country, however, the introduction of a small export subsidy has a different welfare effect in the CES model considered here than in the MO model as analyzed in Bagwell and Lee (2018). This difference arises because of the distinct entry-externality findings across the two models. In both models, the introduction of a small export subsidy entails a tradeoff for the subsidizing country: a subsidy expense is incurred but the Metzler paradox ensures that consumer surplus is increased. The entry-externality effect, however, is always positive in the CES model considered here whereas the sign of the entry-externality effect depends on model parameters in the MO model that we analyze in Bagwell and Lee (2018). Accordingly, we may expect that the unilateral appeal of a small export subsidy is greater in the CES model considered here. We confirm this expectation in the following proposition, which shows for the CES model that in fact the introduction of a small export subsidy is *always* attractive to the subsidizing country:

Proposition 5 *(small export subsidy) For countries l and h with l, $h \in \{H, F\}$ and $l \neq h$, if both countries initially adopt a policy of free trade, then the introduction of a small export subsidy by country l has the following effects: 1). It generates a welfare gain for country l,*

$$\frac{dU^l}{d\tilde{t}^l}\,|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \;\; = \left(\frac{dCS^l}{d\tilde{t}^l} + EXP^l\right)|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \;\; < 0. \tag{66}$$

2). It generates a welfare loss for country h, as indicated in (65).

The finding in Propostion 5 for the CES model differs importantly from the counterpart finding in the MO model that we analyze in Bagwell and Lee (2018). Specifically, the sign of (66) does not depend on model parameters in the CES model, whereas the sign of the corresponding expression in the MO model depends on model parameters related to the sign of entry-externality effect.

Propostion 5 provides a partial perspective on the prohibition on export subsidies in the WTO. According to this proposition, in the CES model, if countries start at global free trade, then a country always has a unilateral incentive to introduce a small export subsidy and moreover the subsidy is a beggar-thy-neighbor policy in that the welfare of the other country is reduced. We confirm in Proposition 7 below, however, a small export subsidy always raises total welfare in the CES model, so that an efficiency-based rationale for the prohibition of export subsidies is not provided by the CES model.

4.3 Introduction of a Small Import and Export Tariff

We consider now a different experiment in which country l unilaterally departs from its free trade policies by simultaneously increasing its import and export tariffs. We show that the introduction of appropriately defined and small increase in country l's import and export tariffs generates a gain for country l and a loss for country h. In this way, we identify a complementary relationship between country l's trade-policy instruments. This relationship is not apparent in the unilateral variations considered above, where the introduction of only one trade policy at a time is considered.

Formally, we now consider the case of small and simultaneous unilateral increases in country l's import and export tariffs, starting at global free trade, where the tariff changes maintain the value of φ_D^{l*} and thus satisfy

$$\frac{\partial \tilde{t}^l}{\partial t^l} \mid_{t^h=\tilde{t}^h=t^l=\tilde{t}^l=0,\bar{\varphi}_D^{l*}} = -\frac{\frac{\partial \varphi_D^{l*}}{\partial t^l}}{\frac{\partial \varphi_D^{l*}}{\partial \tilde{t}^l}} \mid_{t^h=\tilde{t}^h=t^l=\tilde{t}^l=0,\bar{\varphi}_D^{l*}} = \tau^{-k}\left(\frac{f_X}{f_D}\right)^{-\frac{1+k-\sigma}{\sigma-1}} > 0, \tag{67}$$

where the exact expression in (67) uses (18) and derivations found in the proof of Proposition 1 (namely, (71) and (74)). Note that such a change is sure to raise tariff revenue and thus income in country l, since by (43)

$$\frac{dTR^l}{dt^l} \mid_{t^l=\tilde{t}^l=0} = IMP^l > 0 \text{ and } \frac{dTR^l}{d\tilde{t}^l} \mid_{t^l=\tilde{t}^l=0} = EXP^l > 0. \tag{68}$$

Given (28) and (50), it is clear that tariff changes that preserve φ_D^{l*} affect country-l welfare only through tariff revenue.

Consider next the tariff changes that would instead preserve the value of φ_D^{h*} :

$$\frac{\partial \tilde{t}^l}{\partial t^l} \mid_{t^h=\tilde{t}^h=t^l=\tilde{t}^l=0,\bar{\varphi}_D^{h*}} = -\frac{\frac{\partial \varphi_D^{h*}}{\partial t^l}}{\frac{\partial \varphi_D^{h*}}{\partial \tilde{t}^l}} \mid_{t^h=\tilde{t}^h=t^l=\tilde{t}^l=0,\bar{\varphi}_D^{h*}} = \tau^{k}\left(\frac{f_X}{f_D}\right)^{\frac{1+k-\sigma}{\sigma-1}} > 0, \tag{69}$$

where the exact expression in (69) uses (18) and derivations found in the proof of Proposition 1 (namely, (72) and (73)). Given (28) and (50) and starting at global free trade, it is clear that tariff changes that preserve φ_D^{h*} woud not alter country-h welfare.

Using (67) and (68), we see that, starting at global free trade, the introduction of a small import tariff and a small export tariff by country l that satisfies (67) is sure to increase country l's welfare. Since $\tau^k\left(\frac{f_X}{f_D}\right)^{\frac{1+k-\sigma}{\sigma-1}} > \tau^{-k}\left(\frac{f_X}{f_D}\right)^{-\frac{1+k-\sigma}{\sigma-1}}$ holds under our assumption as captured by (38) and (40), we may conclude from (67) and (69) that, for a given increase in t^l, the increase in $\tilde{t}^l$ that maintains φ_D^{l*} is not sufficiently great to maintain φ_D^{h*}. Thus, county h's welfare decreases with respect to this policy change, due to the induced fall in φ_D^{h*} and associated rise in P^h.

We may summarize our findings with the following propoposition:

Proposition 6 *(small import and export tariffs) For countries l and h with l, $h \in \{H, F\}$ and $l \neq h$, if both countries initially adopt a policy of free trade, then the introduction of* *a small import tariff and a small export tariff by country l that satisfies (67) is sure to increase country l's welfare and lower country h's welfare.*

5 Efficient and Nash Trade Policies

In this section, we characterize efficient and Nash tariffs.

5.1 Efficient Symmetric Trade Policies

To characterize efficient tariffs, we begin by establishing that joint welfare depends on tariffs only through the overall barriers to trade, χ^H and χ^F.

Lemma 1 *Joint welfare, $U \equiv U^H + U^F$, depends on individual tariffs, $\{t^H, \tilde{t}^H, t^F, \tilde{t}^F\}$, only through $\chi^H = \frac{1+t^H}{1-\tilde{t}^F}$ and $\chi^F = \frac{1+t^F}{1-\tilde{t}^H}$.*

It follows that countries can effect lump-sum transfers through tariff changes that maintain $\chi^H = \frac{1+t^H}{1-\tilde{t}^F}$ and $\chi^F = \frac{1+t^F}{1-\tilde{t}^H}$. For example, consider an increase in t^l that is balanced against a decrease in $\tilde{t}^h$ so as to keep χ^l constant. With χ^l and χ^h thus unaltered, we know from Lemma 1 that $U^l + U^h$ is unaltered. With P^l and EXP^l also unaltered, the impact of the described change in tariffs on U^l derives from the induced change in $t^l \cdot IMP^l$. We can easily show, however, that the described change raises $t^l \cdot IMP^l$ and thus U^l.[17] With $U^l + U^h$ unaltered, it thus follows that the described change induces a lump-sum transfer from country h to country l.

Since countries can effect lump-sum transfers through adjustments in tariffs, efficient tariffs maximize the sum of the two countries' welfare functions, $U = U^H + U^F$. Using Lemma 1, and following Bagwell and Lee (2018), we can define efficient tariffs using a two-step process. First, we solve the following program,

$$\max_{\{\chi^H,\chi^F\}} U(\chi^H, \chi^F),$$

and thereby determine the overal trade barriers, χ^H and χ^F, that maximize joint welfare, $U(\chi^H, \chi^F)$. Second, we then define the set of *efficient tariffs* by the set of underlying tariffs, $\{t^H, \tilde{t}^H, t^F, \tilde{t}^F\}$, that induce the overall trade barriers that result from the first step. A continuum of tariffs can induce the first-step overall trade barriers.

Let χ^* maximize $U(\chi, \chi)$. This is the trade barrier that maximizes joint welfare under the symmetry constraint that $\chi^H = \chi^F = \chi$. For any value of χ, the tariffs $\{t^H, \tilde{t}^H, t^F, \tilde{t}^F\}$ are *χ-symmetric tariffs* if they induce $\chi^H = \chi^F = \chi$. *Efficient χ-symmetric tariffs* are

[17] To confirm this, we may refer to the second equality in the set-off expression for IMP^l in the proof of Lemma 1 in the Appendix, and observe that the described change induces an increase in $\frac{t^l}{1+t^l}$.

then χ-symmetric tariffs for which $\chi = \chi^*$. Efficient χ-symmetric tariffs are thus efficient within the set of χ-symmetric tariffs. Of course, there is also a continuum of underlying tariffs that induce χ^*.

Following our approach in Bagwell and Lee (2018), we assume that a maximizer χ^* exists that is consistent with the assumptions in Section 2 and interior.[18] We thus assume that (38) holds and hence that (48) and (49) generate a positive number of entrants in each country when the overall barrier is given by $\chi^H = \chi^F = \chi^*$.[19] The meaning of interiority is that χ^* satisfies the first-order condition:

$$\frac{dU(\chi, \chi)}{d\chi}|_{\chi=\chi^*} = 0.$$

We do not maintain an assumption that $U(\chi, \chi)$ is quasi-concave; instead, we introduce this assumption within the propositions below when it is used.

Our next proposition addresses a fundamental question: Is global free trade (i.e., $t^H = \tilde{t}^H = t^F = \tilde{t}^F = 0$) an efficient trade policy?

Proposition 7 *(Free trade and efficiency) If both countries initially adopt a policy of free trade so that $t^H = \tilde{t}^H = t^F = \tilde{t}^F = 0$, then the introduction of a small increase in any tariff lowers joint welfare.*

For the CES model, Proposition 7 indicates that, starting at global free trade, the introduction of a small increase in any tariff lowers joint welfare. In fact, we see from (80) in the proof that the reduction of joint welfare is the same, regardless of which tariff is introduced:

$$\frac{dU}{dt^l}|_{t^l=t^h=\tilde{t}^l=\tilde{t}^h=0} = \frac{dU}{d\tilde{t}^l}|_{t^l=t^h=\tilde{t}^l=\tilde{t}^h=0} < 0. \tag{70}$$

By comparison, in the MO model that we study in Bagwell and Lee (2018), the sign of (70) relates to the sign of the entry-externality effect, which is in turn determined by parameters. Since the entry-externality effect is always positive in the CES model, the unilateral introduction of an export or import subsidy always raises joint welfare in the CES model, as (70) confirms. In this sense, the CES model thus does not offer an efficiency-based rationale for the WTO's prohibition on the use of export subsidies.

Proposition 7 also provides a more complete perspective for the finding in Proposition 5 that, starting at global free trade, the intervening country always gains from the introduction of a small export subsidy. From Proposition 7, we see that such a policy indeed always increases joint welfare and thus the overall global "pie" given the CES model

[18] For convenience, we recall these assumptions here as part of our discussion of efficient χ-symmetric tariffs. We note, however, that these assumptions do not play a role in the next two propositions.

[19] Recall that $N_E^l = N_E^h > 0$ is assured by (38) when tariffs are such that $\chi^l = \chi^h$.

considered here. Thus, in the CES model, the introduction of a small export subsidy is always attractive for the intervening country, since it enables that country to enjoy a larger share of a larger pie.

The next proposition makes a similar point, but starts with any tariffs that achieve free trade in the sense that the overall trade barrier along each channel is zero (i.e., $\chi^H = \chi^F = 1$). The case in which countries start at global free trade is one example, but more generally there is a continuum of tariffs that deliver $\chi^H = \chi^F = 1$.

Proposition 8 *(Free trade and efficiency under χ-symmetric tariffs) If the two countries initially adopt tariffs that achieve free trade so that $\chi^H = \chi^F = 1$, then the introduction of small tariff changes that induce a small and symmetric increase in $\chi = \chi^H = \chi^F$ lowers joint welfare.*

Propositions 7 and 8 have similar implications for trade-agreement design, with the difference being that Proposition 8 allows for a larger set of initial tariffs and then considers symmetric adjustments in the overall trade barrier. In line with our analysis of the MO model in Bagwell and Lee (2018), a specific implication of Proposition 8 is that global free trade is not in general an efficient trade policy, even within the restricted class of χ-symmetric tariffs. A novel implication of Proposition 8 is that, starting with policies in which the overall trade barrier is absent ($\chi^H = \chi^F = 1$), efficiency always would be enhanced in the CES model studied here if trade policies were adjusted to introduce a small and symmetric subsidy to trade ($\chi^H = \chi^F < 1$). By contrast, in the MO model, the sign of $\frac{dU(\chi,\chi)}{d\chi}|_{\chi=1}$ depends on parameters. The distinct implications of the two models in this regard track back to the different entry-externality effects that arise in the models, as captured in Proposition 3 and the discussion following that proposition.

In the proof of Proposition 8, we show that, starting at χ-symmetric tariffs and for χ values consistent with positive entry, a small and symmetric increase in χ lowers the cut-off productivity level in each country, $\varphi_D^{H*} = \varphi_D^{F*}$. As a result of the consequent increase in the price index, consumer surplus falls in each country. Thus, for instance, while an increase in a country's import tariff raises its cut-off productivity level and thereby raises its consumer surplus, symmetric increases in import tariffs that maintain $\chi = \chi^H = \chi^F$ lower each country's cut-off productivity level and thus generate a lower consumer surplus for each country. We derive a similar finding in Bagwell and Lee (2008) for the MO model.[20]

[20] Melitz and Ottaviano (2008, p. 309) also offer a similar finding in the context of a (symmetric) change in the trade cost, τ.

5.2 Nash Trade Policies and Liberalization Paths

We consider now Nash tariffs. A Nash equilibrium is defined as a set of tariffs, $\{t^H, \tilde{t}^H, t^F, \tilde{t}^F\}$, that simultaneously solves

$$\max_{t^l, \widetilde{t}^l} U^l \text{ for } l = H, F,$$

where we recall that $t^l > -1$ and $\widetilde{t}^l < 1$. *Nash tariffs* are then a set of tariffs that form a Nash equilibrium. A *symmetric Nash equilibrium* is a Nash equilibrium such that $t^H = t^F$ and $\widetilde{t}^H = \widetilde{t}^F$. Given a symmetric Nash equilibrium, the associated *symmetric Nash tariffs* form the pair $(t^N, \widetilde{t}^N)$, where $t^N \equiv t^H = t^F$ is the symmetric Nash import tariff and $\widetilde{t}^N \equiv \widetilde{t}^H = \widetilde{t}^F$ is the symmetric Nash export tariff. Symmetric Nash tariffs are clearly χ-symmetric tariffs. The associated symmetric Nash value for χ is given as $\chi^N \equiv (1 + t^N)/(1 - \widetilde{t}^N)$.

Following our approach in Bagwell and Lee (2018), we assume that there exists a symmetric Nash equilibrium that is consistent with the assumptions in Section 2 and interior. We thus assume that $t^N > -1$, $\widetilde{t}^N < 1$, (38) holds and hence that (48) and (49) generate a positive number of entrants in each country when the overall barrier is given by $\chi^H = \chi^F = \chi^N$.[21] The meaning of interiority is that t^N and $\widetilde{t}^N$ satisfy the associated first-order conditions:

$$\frac{dU^l}{dt^l}|_{t^l=t^h=t^N, \tilde{t}^l=\tilde{t}^h=\tilde{t}^N} = \frac{dU^l}{d\widetilde{t}^l}|_{t^l=t^h=t^N, \tilde{t}^l=\tilde{t}^h=\tilde{t}^N} = 0 \text{ for } l = H, F.$$

Our next result provides a condition under which the symmetric Nash equilibrium is inefficient with an overall trade barrier that is higher than efficient. This result in turn enables us to identify efficiency-enhancing liberalization paths.

Proposition 9 *(Nash, efficiency and liberalization paths) If $U(\chi, \chi)$ is quasi-concave in χ, then the symmetric Nash equilibrium is inefficient with a value for χ that is too high: $\chi^N > \chi^*$. Starting at the symmetric Nash equilibrium, countries thus mutually gain by symmetrically exchanging small reductions in import tariffs, export tariffs, or combinations thereof.*

An interesting implication of Proposition 9 is that a small and symmetric reduction in export tariffs generates mutual gains, even though an export tariff reduction by one country imposes a terms-of-trade loss on its trading partner.[22] Of course, if the symmetric Nash equilibrium is characterized by the use of export subsidies (i.e., negative export tariffs), then Proposition 9 provides that countries can enjoy mutual gains by exchanging

[21] Recall that $N_E^l = N_E^h > 0$ is assured by (38) when tariffs are such that $\chi^l = \chi^h$.

[22] For further discussion, see Bagwell and Staiger (2012) and Bagwell and Lee (2018), which provide related findings for the linear Cournot delocation and MO models, respectively.

small and symmetric increases in their export subsidies. More generally, Proposition 9 provides conditions under which the symmetric Nash equilibrium is inefficient, with an overall trade barrier that is too high ($\chi^N > \chi^*$); accordingly, it provides a possible interpretation for why early GATT rounds emphasized negotiated reductions in import tariffs while treating export subsidies in a more permissive way.[23] Moreover, as Proposition 7 indicates, for the CES model and in contrast to the MO model that we analyze in Bagwell and Lee (2018), an efficiency-based rationale for a restriction against the unilateral introduction of a small export subsidy fails to emerge even after governments have achieved through negotiations an outcome sufficiently close to global free trade.

We now combine findings from the preceding two propositions as follows:

Proposition 10 *(Nash and efficient tariffs) Assume that $U(\chi, \chi)$ is quasi-concave in χ. Then $\chi^N > \chi^*$ and $1 > \chi^*$.*

The finding that $\chi^N > \chi^*$ follows directly from Proposition 9, and the finding that $1 > \chi^*$ follows from Proposition 8 under the quasi-concavity of $U(\chi, \chi)$ in χ. Thus, under quasi-concavity, the overall trade barrier is too high at the symmetric Nash equilibrium and, indeed, even at global free trade.

5.3 Numerical Example

We now offer a brief numerical analysis of the CES model. We specify the following parameter values: $k = 2$, $f_e = 0.1, \tau = 5, \sigma = 1.5, \theta = 0.1, f_X = 3$ and $f_D = 1.5$. These parameter values satisfy our restrictions, since

$$\begin{aligned}
\sigma(1-\theta) - 1 &= 0.35 > 0 \\
1 + k - \sigma &= 1.5 > 0 \\
1 - \frac{f_X}{f_D} A(\chi)^{-k} &> 0 \text{ for } \chi \in \{\chi^*, \chi^N, 1\} \\
N_E(\chi, \chi) &> 0 \text{ for } \chi \in \{\chi^*, \chi^N, 1\} \\
\varphi_X^*(\chi, \chi) &> \varphi_D^*(\chi, \chi) > 1 \text{ for } \chi \in \{\chi^*, \chi^N, 1\}
\end{aligned}$$

For this specification, we find that $\chi^* = 0.475 < 0.548 = \chi^N$. The symmetric Nash tariffs that generate χ^N are given by $t^N = 0.089$ and $\widetilde{t}^N = -0.987$. Thus, in this example, the symmetric Nash tariffs entail an import tariff and an export subsidy. We can also verify that $q_0^l > 0$ for $l = H, F$ for $\chi \in \{\chi^*, \chi^N, 1\}$ under the further restriction that, for each χ value in this set, the two countries adopt symmetric import tariffs and symmetric export tariffs, respectively. Thus, for example, $q_0^l > 0$ for $l = H, F$ holds at the symmetric Nash

[23] As Sykes (2005) discusses, GATT restrictions on export subsidies tightened over time.

tariffs.[24] We can verify for this specification that the function $U(\chi, \chi)$ is quasi-concave for tariffs consistent with the assumptions in Section 2.

The numerical findings align with our propositions. As Proposition 10 indicates, the overall trade barrier is too high at the symmetric Nash equilibrium and, indeed, even at global free trade: $\chi^N > \chi^*$ and $1 > \chi^*$. For this specification, we find further that the overall trade barrier in the symmetric Nash equilibrium entails a trade subsidy: $1 > \chi^N$.

6 Conclusion

We analyze trade policy in a symmetric, two-country model with heterogeneous firms, monopolistic competition and a freely traded outside good. The model features a quasi-linear utility function, where differentiated varieties are aggregated according to a CES preference function and the homogeneous outside good enters in a linear and additive fashion. Letting each country's objective be represented by its national welfare and assuming that each country has available ad valorem import and export tariffs (or subsidies), we characterize unilateral policy interventions that raise the welfare of the intervening country and harm its trading partner, efficient trade policies that maximize the joint welfare of the two countries, and Nash trade policies.

Many of our findings are similar to those in Bagwell and Lee (2018), where we follow Meltiz and Ottaviano (2008) and consider a heterogeneous-firms model in which consumers have quadratic preferences for the differentiated sector. An important difference across the two models, however, concerns the efficiency properties of the market equilibrium for a closed-economy benchmark setting. In the CES model, the market always provides too little entry in the differentiated sector, whereas insufficient entry obtains in the model of Melitz and Ottaviano only for a subset of the parameter space. This difference leads to different results concerning the characterization of efficient symmetric tariffs and their relationship to free trade and to the symmetric Nash tariffs. The difference also underlies novel implications regarding the treatment of export subsidies in the WTO. For tariffs that are sufficiently close to global free trade, Bagwell and Lee (2018) find that the WTO prohibition on export subsidies can be given an efficiency-based rationale for a subset of parameters. For the CES model considered in this paper, however, the WTO prohibition on export subsidies fails to obtain a similar efficiency-based rationale.

Our analysis can be extended in several directions. One particularly interesting direction would be to extend the analysis to consider multiple countries and study the

[24] Recall that q_0^l depends on tariffs directly and not just through the associated values for the overall trade barriers. Even when $\chi = \chi^l = \chi^h$, if we seek to evaluate the sign of q_0^l, we must specify the tariffs that induce χ^l and χ^h. A symmetry restriction under which the two countries select a common import tariff and also a common export tariff generates a natural point for evaluation.

trade-diversion effects of bilateral liberalization in the context of a model with endogenous firm selection.

7 Appendix

Proof of Proposition 1: To establish (51), we use (18) and (37) to calculate that

$$\frac{\partial \varphi_D^{l*}}{\partial t^l} = \left(\frac{\varphi_D^{l*}(\frac{f_X}{f_D})^2}{A^l(A^l A^h)^k[1 - \left(\frac{f_X}{f_D}\right)^2 \cdot (A^l \cdot A^h)^{-k}]} \right) \frac{\partial A^l}{\partial \chi^l} \frac{\partial \chi^l}{\partial t^l} > 0, \tag{71}$$

where the first term indicates how φ_D^{l*} varies with A^l and is positive for the tariffs under consideration by (38) and the implied (39) and where $\frac{\partial A^l}{\partial \chi^l} > 0$ and $\frac{\partial \chi^l}{\partial t^l} > 0$ are easily confirmed. Since $\frac{\partial \chi^l}{\partial \tilde{t}^h} > 0$, we may similarly confirm that

$$\frac{\partial \varphi_D^{l*}}{\partial \tilde{t}^h} = \left(\frac{\varphi_D^{l*}(\frac{f_X}{f_D})^2}{A^l(A^l A^h)^k[1 - \left(\frac{f_X}{f_D}\right)^2 \cdot (A^l \cdot A^h)^{-k}]} \right) \frac{\partial A^l}{\partial \chi^l} \frac{\partial \chi^l}{\partial \tilde{t}^h} > 0. \tag{72}$$

Likewise, we may use (18) and (37) to calculate that

$$\frac{\partial \varphi_D^{l*}}{\partial t^h} = \left(\frac{-\varphi_D^{l*}(\frac{f_X}{f_D})[1 - \left(\frac{f_X}{f_D}\right) \cdot \left(A^l\right)^{-k}]}{(A^h)^{k+1}[1 - \left(\frac{f_X}{f_D}\right)^2 \cdot (A^l \cdot A^h)^{-k}][1 - \left(\frac{f_X}{f_D}\right) \cdot (A^h)^{-k}]} \right) \frac{\partial A^h}{\partial \chi^h} \frac{\partial \chi^h}{\partial t^h} < 0, \tag{73}$$

where the first term indicates how φ_D^{l*} varies with A^h and is negative for the tariffs under consideration by (38) and the implied (39) and where as before $\frac{\partial A^h}{\partial \chi^h} > 0$ and $\frac{\partial \chi^h}{\partial t^h} > 0$. Since $\frac{\partial \chi^h}{\partial \tilde{t}^l} > 0$, we may similarly confirm that

$$\frac{\partial \varphi_D^{l*}}{\partial \tilde{t}^l} = \left(\frac{-\varphi_D^{l*}(\frac{f_X}{f_D})[1 - \left(\frac{f_X}{f_D}\right) \cdot \left(A^l\right)^{-k}]}{(A^h)^{k+1}[1 - \left(\frac{f_X}{f_D}\right)^2 \cdot (A^l \cdot A^h)^{-k}][1 - \left(\frac{f_X}{f_D}\right) \cdot (A^h)^{-k}]} \right) \frac{\partial A^h}{\partial \chi^h} \frac{\partial \chi^h}{\partial \tilde{t}^l} < 0. \tag{74}$$

To establish (52), we recall from (18) that $\varphi_X^{l*} = A^h \cdot \varphi_D^{h*}$, where A^h in turn depends on tariffs only through χ^h. It now follows immediately from (73) that

$$\frac{\partial \varphi_X^{l*}}{\partial t^l} = A^h \cdot \frac{\partial \varphi_D^{h*}}{\partial t^l} < 0$$

and similarly that $\frac{\partial \varphi_X^{l*}}{\partial \tilde{t}^h} = A^h \cdot \frac{\partial \varphi_D^{h*}}{\partial \tilde{t}^h} < 0$. Next, we find that

$$\frac{\partial \varphi_X^{l*}}{\partial t^h} = \left(\frac{\varphi_D^{h*}}{1 - \left(\frac{f_X}{f_D}\right)^2 \cdot (A^l \cdot A^h)^{-k}} \right) \frac{\partial A^h}{\partial \chi^h} \frac{\partial \chi^h}{\partial t^h} > 0,$$

where the first term indicates how φ_X^{l*} varies with A^h and is positive for the tariffs under consideration by (38) and the implied (39) and where $\frac{\partial A^h}{\partial \chi^h} > 0$ and $\frac{\partial \chi^h}{\partial t^h} > 0$ follow as before. Since $\frac{\partial \chi^h}{\partial \tilde{t}^l} > 0$, we may similarly confirm that $\frac{\partial \varphi_x^{l*}}{\partial \tilde{t}^l} > 0$.

Proof of Proposition 3: To begin, we find it convenient to define and characterize the elasticity of the productivity cut-off level with respect to the level of entry

$$\epsilon_{\varphi^*, N_E} \equiv \frac{d\ln(\varphi^*)}{d\ln(N_E)} = \frac{\frac{1}{\sigma-1}}{\frac{1+k-\sigma}{\sigma-1} + \frac{(\sigma-1)(1-\theta)}{\sigma(1-\theta)-1}} > 0,$$

where the characterization follows easily from (60).

Referring to (61), we may calculate $\frac{dCS}{dN_E}$ as

$$\frac{dCS}{dN_E} = \frac{dCS}{dP}\frac{dP}{d\varphi^*}\frac{d\varphi^*}{dN_E}.$$

Using (59), (61) and that $\frac{d\varphi^*}{dN_E} = \frac{\varphi^*}{N_E} \cdot \epsilon_{\varphi^*, N_E}$, straightforward calculations yield

$$\frac{dCS}{dN_E} = \frac{(\sigma-1)(1-\theta)}{\sigma(1-\theta)-1} \cdot \frac{(P)^{-\frac{\theta}{1-\theta}}}{N_E} \cdot \epsilon_{\varphi^*, N_E}, \tag{75}$$

so that $\frac{dCS}{dN_E} > 0$ for $N_E = N_E^m > 0$.[25]

Similarly, we may refer to (54) and calculate $\frac{d\bar{\pi}}{dN_E}$ as

$$\frac{d\bar{\pi}}{dN_E} = \frac{d\bar{\pi}}{d\varphi^*}\frac{d\varphi^*}{dN_E}.$$

Using (54) and that $\frac{d\varphi^*}{dN_E} = \frac{\varphi^*}{N_E} \cdot \epsilon_{\varphi^*, N_E}$, straightforward calculations give

$$\frac{d\bar{\pi}}{dN_E} = -k \cdot \frac{\bar{\pi}}{N_E} \cdot \epsilon_{\varphi^*, N_E}, \tag{76}$$

so that $\frac{d\bar{\pi}}{dN_E} < 0$ for $N_E = N_E^m > 0$, where we recall that at the free-entry solution $\bar{\pi} = f_e > 0$.

We now combine (75) and (76) to evaluate EXT as defined in (62) at $N_E = N_E^m > 0$. We find that

$$\left(\frac{dCS}{dN_E} + N_E\frac{d\bar{\pi}}{dN_E}\right)|_{N_E=N_E^m} = \left([\frac{(\sigma-1)(1-\theta)}{\sigma(1-\theta)-1} \cdot \frac{(P)^{-\frac{\theta}{1-\theta}}}{N_E} - k\bar{\pi}] \cdot \epsilon_{\varphi^*, N_E}\right)|_{N_E=N_E^m}.$$

[25] Note that at $N_E = N_E^m > 0$, we have that $\bar{\pi} = f_e > 0$. We may thus conclude from (54) that the associated value for φ^* satisfies $\varphi^* > 0$. We then have from (59) that the associated value for P satisfies $P > 0$.

Thus, since $\epsilon_{\varphi^*,N_E} > 0$, we have that

$$sign\left(\frac{dCS}{dN_E} + N_E\frac{d\bar{\pi}}{dN_E}\right)|_{N_E=N_E^m} = sign\left([\frac{(\sigma-1)(1-\theta)}{\sigma(1-\theta)-1}\cdot\frac{(P)^{-\frac{\theta}{1-\theta}}}{N_E} - k\bar{\pi}]\right)|_{N_E=N_E^m}. \tag{77}$$

To evaluate the sign of the RHS of (77), we refer to (58) and (59) and observe that

$$\frac{(P)^{-\frac{\theta}{1-\theta}}}{N_E} = \bar{r} = (\varphi^*)^{-k}(\frac{k}{1+k-\sigma})\sigma \cdot f_D.$$

Similarly, we have from (54) that

$$k\bar{\pi} = k\left(\varphi^*\right)^{-k}\frac{(\sigma-1)\,f_D}{1+k-\sigma}.$$

Substituting these expressions into the RHS of (77) and simplifying, we find that

$$sign\left(\frac{dCS}{dN_E} + N_E\frac{d\bar{\pi}}{dN_E}\right)|_{N_E=N_E^m} = sign\left((\varphi^*)^{-k}\cdot\frac{(\sigma-1)}{\sigma(1-\theta)-1}\cdot\frac{kf_D}{1+k-\sigma}\right)|_{N_E=N_E^m} > 0,$$

where the inequality follows since $\varphi^* > 0$ at $N_E = N_E^m$.[26]

Proof of Proposition 5: The proof of part 2) is already established in (65). Consider then part 1) of the proposition. Using (50), we observe that

$$\frac{dU^l}{d\tilde{t}^l}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \quad = \left(\frac{dCS^l}{d\tilde{t}^l} + EXP^l\right)|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0},$$

and we compute each term in turn.

Starting with the second term, we use (42) and find that

$$EXP^l\,|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} = \left(\frac{N_E^l}{1-\tilde{t}^l}\left(\varphi_X^{l*}\right)^{-k}\frac{k\cdot\sigma\cdot f_X}{1+k-\sigma}\right)|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0}.$$

We now recall from (18), (28) and (37) that $P^l = P^h$ when $\chi^l = \chi^h$. Using as well (18) and (49) and the definitions of T_1^l and T_2^l in (48), we find that

$$EXP^l\,|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} = \left(\frac{\left(P^{FT}\right)^{-\frac{\theta}{1-\theta}}\frac{f_X}{f_D}}{\frac{f_X}{f_D} + (A^{FT})^k}\right), \tag{78}$$

where $P^{FT} \equiv P^l\,|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0}$ and $A^{FT} \equiv A^l\,|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0}$.

[26] See footnote 25 for confirmation.

Turning next to the first term, we have that

$$\frac{dCS^l}{d\tilde{t}^l}\mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \quad = \frac{dCS^l}{dP^l}\frac{dP^l}{d\varphi_D^{l*}}\frac{d\varphi_D^{l*}}{d\tilde{t}^l}\mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} .$$

Using (7), (28) and (74), we find

$$\frac{dCS^l}{d\tilde{t}^l}\mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0}= -\left(\frac{(1-\theta)\sigma}{\sigma(1-\theta)-1}\right)\left(\frac{\left(P^{FT}\right)^{-\frac{\theta}{1-\theta}}\frac{f_X}{f_D}}{[1-(\frac{f_X}{f_D})^2\cdot(A^{FT})^{-2k}](A^{FT})^k}\right). \tag{79}$$

Finally, we add (78) and (79) to obtain that

$$\begin{aligned}\frac{dU^l}{d\tilde{t}^l} \quad \mid \quad & _{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} = \left(\frac{dCS^l}{d\tilde{t}^l}+EXP^l\right)\mid_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \quad = \\ & \left(\frac{\left(P^{FT}\right)^{-\frac{\theta}{1-\theta}}\frac{f_X}{f_D}}{[1+\frac{f_X}{f_D}(A^{FT})^{-k}](A^{FT})^k}\right)\left(1-(\frac{(1-\theta)\sigma}{\sigma(1-\theta)-1})(\frac{1}{1-\frac{f_X}{f_D}(A^{FT})^{-k}})\right) \\ < \quad & 0,\end{aligned} \tag{80}$$

where the inequality follows since $\frac{(1-\theta)\sigma}{\sigma(1-\theta)-1} > 1$, $A^{FT} = \tau\left(\frac{f_X}{f_D}\right)^{\frac{1}{\sigma-1}} > 0$ by (18), and $1-\frac{f_X}{f_D}(A^{FT})^{-k} \in (0,1)$ by (38) and (40).

Proof of Lemma 1: Using (7) and (50), we have that

$$U^l + U^h = 2 + CS^l + CS^h + \left(t^l + \tilde{t}^h\right)\cdot IMP^l + \left(\tilde{t}^l + t^h\right)\cdot EXP^l, \tag{81}$$

where we use from (41) and (42) that $IMP^l = EXP^h$. We know from (7) that CS^l is a function of P^l, and we see from (28) that P^l is a function of φ_D^{l*}. In turn, we have from (37) and the definition of A^l in (18) that φ_D^{l*} depends on tariffs only through χ^l and χ^h. Thus, P^l is a function of tariffs only through χ^l and χ^h. Next, using (18), (41) and (42), we have that

$$\begin{aligned}IMP^l &= \frac{N_E^h}{1-\tilde{t}^h}\left(A^l\cdot\varphi_D^{l*}\right)^{-k}\frac{k\cdot\sigma\cdot f_X}{1+k-\sigma} = \frac{N_E^h}{1+t^l}\chi^l\left(A^l\cdot\varphi_D^{l*}\right)^{-k}\frac{k\cdot\sigma\cdot f_X}{1+k-\sigma} \\ EXP^l &= \frac{N_E^l}{1-\tilde{t}^l}\left(A^h\cdot\varphi_D^{h*}\right)^{-k}\frac{k\cdot\sigma\cdot f_X}{1+k-\sigma} = \frac{N_E^l}{1+t^h}\chi^h\left(A^h\cdot\varphi_D^{h*}\right)^{-k}\frac{k\cdot\sigma\cdot f_X}{1+k-\sigma}.\end{aligned}$$

As above, we have from (18) that A^l depends on tariffs only through χ^l and that φ_D^{l*} depends on tariffs only through χ^l and χ^h. Referring to (49), we may similarly argue that

N_E^l depends on tariffs only through χ^l and χ^h. Now observe that

$$\frac{t^l + \tilde{t}^h}{1 + t^l} = \frac{\chi^l - 1}{\chi^l}.$$

Using this relationship along with the final equalities in the expressions for IMP^l and EXP^l just provided, we may now conclude that $U^l + U^h$ depends on tariffs only through χ^l and χ^h.

Proof of Proposition 7: Using (81), we find that

$$\begin{aligned} \frac{dU}{dt^l}|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0} &= [\frac{dCS^l}{dt^l} + \frac{dCS^h}{dt^l} + IMP^l]|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0} \\ \frac{dU}{d\tilde{t}^l}|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0} &= [\frac{dCS^l}{d\tilde{t}^l} + \frac{dCS^h}{d\tilde{t}^l} + EXP^l]|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0}. \end{aligned} \tag{82}$$

We must show that both joint-welfare derivatives in (82) are negative.

As argued in the proof of Lemma 1, P^l is a function of tariffs only through χ^l and χ^h; thus, it follows from (7) that CS^l depends on a tariffs only through χ^l and χ^h. Given that the derivative of χ^l with respect to t^l and $\tilde{t}^h$ when evaluated at global free trade is unity, and exploiting symmetry, we have that

$$\begin{aligned} \frac{dCS^l}{dt^l}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} &= \frac{dCS^l}{d\tilde{t}^h}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} = \frac{dCS^l}{d\chi^l}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} = \\ \frac{dCS^h}{d\chi^h}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} &= \frac{dCS^h}{dt^h}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} = \frac{dCS^h}{d\tilde{t}^l}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0}. \end{aligned} \tag{83}$$

Likewise, we have that

$$\begin{aligned} \frac{dCS^l}{d\tilde{t}^l}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} &= \frac{dCS^l}{dt^h}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} = \frac{dCS^l}{d\chi^h}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} = \\ \frac{dCS^h}{d\chi^l}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} &= \frac{dCS^h}{d\tilde{t}^h}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} = \frac{dCS^h}{dt^l}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0}. \end{aligned} \tag{84}$$

These relationships imply that, starting at global free trade, the implication of any tariff change for consumer surplus in any country is known once we calculate $\frac{dCS^l}{dt^l}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0}$ and $\frac{dCS^l}{d\tilde{t}^l}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0}$.

Using (7), (28) and (71), we find that

$$\frac{dCS^l}{dt^l}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} = \frac{dCS^l}{dP^l}\frac{dP^l}{d\varphi_D^{l*}}\frac{d\varphi_D^{l*}}{dt^l}|_{t^l=\tilde{t}^l=t^h=\tilde{t}^h=0} \tag{85}$$
$$= \left(\frac{(1-\theta)\sigma}{\sigma(1-\theta)-1}\right)\left(\frac{\left(P^{FT}\right)^{-\frac{\theta}{1-\theta}}(\frac{f_X}{f_D})^2}{[1-(\frac{f_X}{f_D})^2\cdot(A^{FT})^{-2k}](A^{FT})^{2k}}\right).$$

Using (84), we have that

$$[\frac{dCS^h}{dt^l}+IMP^l]|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0}=[\frac{dCS^l}{d\tilde{t}^l}+IMP^l]|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0}.$$

We know further that

$$IMP^l|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0} = EXP^h|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0} = EXP^l|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0},$$

where the first equality follows from (41) and (42) and the second equality follows from symmetry.

Using these relationships, and referring to (82), we thus have that

$$\frac{dU}{dt^l}|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0} = [\frac{dCS^l}{dt^l}+\frac{dCS^h}{dt^l}+IMP^l]|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0}$$
$$= [\frac{dCS^l}{dt^l}+\frac{dCS^l}{d\tilde{t}^l}+EXP^l]|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0}.$$

We can now use (85) to substitute for $\frac{dCS^l}{dt^l}|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0}$ and we can likewise use (80) to substitute for $[\frac{dCS^l}{d\tilde{t}^l}+EXP^l]|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0}$. Proceeding in this way, we find that

$$\frac{dU}{dt^l}|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0} = -\left(\frac{1}{\sigma(1-\theta)-1}\right)\left(\frac{\left(P^{FT}\right)^{-\frac{\theta}{1-\theta}}\frac{f_X}{f_D}}{[1+\frac{f_X}{f_D}(A^{FT})^{-k}](A^{FT})^k}\right)<0, \tag{86}$$

where the inequality is assured at global free trade given (38) and (40).

Next, we observe that

$$\frac{dU}{d\tilde{t}^l}|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0} = [\frac{dCS^l}{d\tilde{t}^l}+\frac{dCS^h}{d\tilde{t}^l}+EXP^l]|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0}$$
$$= [\frac{dCS^l}{d\tilde{t}^l}+\frac{dCS^l}{dt^l}+EXP^l]|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0}$$
$$= \frac{dU}{dt^l}|_{t^H=\tilde{t}^H=t^F=\tilde{t}^F=0} < 0,$$

where the first and last equalities use (82), the second equality uses (83), and the inequality follows from (86).

Proof of Proposition 8: To prove this proposition, it is convenient to introduce notation that explicitly captures the dependence of the key functions on $\chi = \chi^H = \chi^F$ when χ-symmetric tariffs are used. To this end, we define

$$\begin{aligned}
A(\chi) &\equiv \tau \cdot (\chi)^{\frac{\sigma}{\sigma-1}} \left(\frac{f_X}{f_D}\right)^{\frac{1}{\sigma-1}} \\
\varphi_D^*(\chi^l, \chi^h) &\equiv \left(\frac{\left(1 - \frac{f_X}{f_D}\left(A(\chi^h)^{-k}\right)\right)\phi}{f_D\left(1 - \left(\frac{f_X}{f_D}\right)^2 \cdot \left(A(\chi^l)\cdot A(\chi^h)\right)^{-k}\right)} \right)^{-1/k} \\
\varphi_X^*(\chi^l, \chi^h) &\equiv A(\chi^l)\cdot \varphi_D^*(\chi^l, \chi^h) \\
P(\chi^l, \chi^h) &\equiv (\sigma \cdot f_D)^{\frac{1-\theta}{\sigma(1-\theta)-1}} \left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi_D^*(\chi^l, \chi^h)}\right)^{\frac{(1-\theta)(\sigma-1)}{\sigma(1-\theta)-1}} \\
CS(\chi^l, \chi^h) &\equiv \left(\frac{1-\theta}{\theta}\right)\left(P(\chi^l, \chi^h)\right)^{-\frac{\theta}{1-\theta}}.
\end{aligned}$$

Recalling that $\chi^l = (1+t^l)/(1-\tilde{t}^h)$, we may use (7), (18) and (28) to confirm that $A(\chi^l) = A^l$, $\varphi_D^*(\chi^l, \chi^h) = \varphi_D^{l*}$, $\varphi_X^*(\chi^l, \chi^h) = \varphi_X^{h*}$, $P(\chi^l, \chi^h) = P^l$ and $CS(\chi^l, \chi^h) = CS^l$.

Similarly, to represent the number of entrants, we may define

$$\begin{aligned}
T_1(\chi^l, \chi^h) &\equiv \chi^l \left(A(\chi^l)\right)^{-k}\left(\varphi_D^*(\chi^l, \chi^h)\right)^{-k} \frac{k\cdot\sigma\cdot f_X}{1+k-\sigma} \\
T_2(\chi^l, \chi^h) &\equiv \left(\varphi_D^*(\chi^l, \chi^h)\right)^{-k} \frac{k\cdot\sigma\cdot f_D}{1+k-\sigma} \\
N_E(\chi^l, \chi^h) &\equiv \frac{T_2(\chi^h, \chi^l)\left(P(\chi^l, \chi^h)\right)^{-\frac{\theta}{1-\theta}} - T_1(\chi^l, \chi^h)\left(P(\chi^h, \chi^l)\right)^{-\frac{\theta}{1-\theta}}}{T_2(\chi^l, \chi^h)\cdot T_2(\chi^h, \chi^l) - T_1(\chi^l, \chi^h)\cdot T_1(\chi^h, \chi^l)}
\end{aligned}$$

where by (48) and (49) we have that $T_1(\chi^l, \chi^h) = T_1^l$, $T_1(\chi^h, \chi^l) = T_1^h$, $T_2(\chi^l, \chi^h) = T_2^l$, $T_2(\chi^h, \chi^l) = T_2^h$ and $N_E(\chi^l, \chi^h) = N_E^l$.

Finally, trade volume may be represented using

$$f(\chi^l, \chi^h) \equiv N_E(\chi^h, \chi^l)(A(\chi^l))^{-k}(\varphi_D^*(\chi^l, \chi^h))^{-k}\frac{k\cdot\sigma\cdot f_X}{1+k-\sigma},$$

where by (18), (41) and (42) it then follows that $IMP^l = EXP^h = \frac{f(\chi^l,\chi^h)}{1-\tilde{t}^h}$. Thus, and as we argue in Bagwell and Lee (2018) for the MO model, we may understand $f(\chi^l, \chi^h)$ as measuring the value of trade into country l when using delivered (consumer) prices.

Referring to the expression for joint welfare given in (81) and using $IMP^l = EXP^h =$

$\frac{f(\chi^l,\chi^h)}{1-\tilde{t}^h}$ along with $\chi^l - 1 = \frac{t^l+\tilde{t}^h}{1-\tilde{t}^h}$, we may use our definitions above to define joint welfare,

$$U(\chi^l,\chi^h) \equiv 2 + \left(\chi^l - 1\right) f(\chi^l,\chi^h) + \left(\chi^h - 1\right) f(\chi^h,\chi^l) + CS(\chi^l,\chi^h) + CS(\chi^h,\chi^l),$$

where $U(\chi^l,\chi^h) = U^l + U^h$. At χ-symmetric tariffs, we have

$$U(\chi,\chi) = 2[1 + (\chi - 1)\, f(\chi,\chi) + CS(\chi,\chi)], \tag{87}$$

and we may use this expression to evaluate efficiency relative to the class of χ-symmetric tariffs.

To this end, we use (87) and observe that

$$\frac{dU(\chi,\chi)}{d\chi} = 2[(\chi-1)\frac{df(\chi,\chi)}{d\chi} + f(\chi,\chi) + \frac{dCS(\chi,\chi)}{d\chi}], \tag{88}$$

and our next step is to characterize the bracketed expression.

We begin with $\frac{dCS(\chi,\chi)}{d\chi}$. We find that

$$\begin{aligned}\frac{dCS(\chi,\chi)}{d\chi} &= [\frac{\partial CS(\chi^l,\chi^h)}{\partial \chi^l} + \frac{\partial CS(\chi^l,\chi^h)}{\partial \chi^h}]|_{\chi=\chi^l=\chi^h} \qquad (89)\\ &= [\left(\frac{\theta(\sigma-1)}{\sigma(1-\theta)-1}\right)\left(\frac{CS(\chi^l,\chi^h)}{\varphi_D^*(\chi^l,\chi^h)}\right)\left(\frac{\partial\varphi_D^*(\chi^l,\chi^h)}{\partial\chi^l} + \frac{\partial\varphi_D^*(\chi^l,\chi^h)}{\partial\chi^h}\right)]|_{\chi=\chi^l=\chi^h},\end{aligned}$$

where calculations confirm that

$$\begin{aligned}\frac{\partial\varphi_D^*(\chi^l,\chi^h)}{\partial\chi^l}|_{\chi=\chi^l=\chi^h} &= \frac{\sigma}{\sigma-1}(\frac{f_X}{f_D})^2\frac{\varphi_D^*(\chi,\chi)}{\chi}\frac{1}{(A(\chi))^{2k} - (\frac{f_X}{f_D})^2}\\ \frac{\partial\varphi_D^*(\chi^l,\chi^h)}{\partial\chi^h}|_{\chi=\chi^l=\chi^h} &= -\frac{1}{(A(\chi))^{-k}}\frac{\sigma}{\sigma-1}\frac{f_X}{f_D}\frac{\varphi_D^*(\chi,\chi)}{\chi}\frac{1}{(A(\chi))^{2k} - (\frac{f_X}{f_D})^2}\end{aligned}$$

so that

$$\left(\frac{\partial\varphi_D^*(\chi^l,\chi^h)}{\partial\chi^l} + \frac{\partial\varphi_D^*(\chi^l,\chi^h)}{\partial\chi^h}\right)|_{\chi=\chi^l=\chi^h} = -(\frac{\sigma}{\sigma-1})\frac{f_X}{f_D}\frac{\varphi_D^*(\chi,\chi)}{\chi}\frac{1}{(A(\chi))^k + \frac{f_X}{f_D}}. \tag{90}$$

Using (90), we can rewrite (89) as

$$\frac{dCS(\chi,\chi)}{d\chi} = -\left(\frac{\theta\sigma}{\sigma(1-\theta)-1}\right)\left(\frac{f_X}{f_D}\frac{CS(\chi,\chi)}{\chi}\frac{1}{(A(\chi))^k + \frac{f_X}{f_D}}\right). \tag{91}$$

Looking at (90) and (91), and for values of χ such that a positive number of entrants occurs, we see that a symmetric increase in the overall trade barrier lowers the cut-off productivity level and thus consumer surplus, as in the MO model that we examine in Bagwell and Lee (2018).

Referring to (88), we have that

$$\frac{dU(\chi,\chi)}{d\chi}|_{\chi=1} = 2[f(1,1) + \frac{dCS(\chi,\chi)}{d\chi}|_{\chi=1}].$$

Using the definitions above and (91), we find that

$$\begin{aligned} f(1,1) &= \frac{f_X}{f_D}\left(\frac{P(1,1)^{-\frac{\theta}{\theta-1}}}{(A(1))^k + \frac{f_X}{f_D}}\right) > 0 \\ \frac{dCS(\chi,\chi)}{d\chi}|_{\chi=1} &= -\frac{f_X}{f_D}\left(\frac{(1-\theta)\sigma}{\sigma(1-\theta)-1}\right)\left(\frac{P(1,1)^{-\frac{\theta}{\theta-1}}}{(A(1))^k + \frac{f_X}{f_D}}\right) < 0, \end{aligned}$$

where the inequality is assured in the absence of trade barriers by (38) and (40). Using these expressions, simple calculations now give that

$$\frac{dU(\chi,\chi)}{d\chi}|_{\chi=1} = -2\left(\frac{1}{\sigma(1-\theta)-1}\right)\left(\frac{P(1,1)^{-\frac{\theta}{1-\theta}}\frac{f_X}{f_D}}{[1+\frac{f_X}{f_D}(A(1))^{-k}](A(1))^k}\right) < 0, \tag{92}$$

which completes the proof. We note that (92) is consistent with (86), given $A(1) = A^{FT}$ and $P(1,1) = P^{FT}$ and with the "2" in (92) reflecting that χ^l and χ^h are both changed in the experiment that leads to (92).

Proof of Proposition 9: The proof is similar to that given in Bagwell and Lee (2018) for the MO model. We begin by considering country l's welfare. Referring to (50) and using the definitions developed in the proof of Proposition 8, we can re-write country l's welfare as

$$U^l = 1 + \frac{t^l}{1-\tilde{t}^h}\cdot f(\chi^l,\chi^h) + \frac{\tilde{t}^l}{1-\tilde{t}^l}\cdot f(\chi^h,\chi^l) + CS(\chi^l,\chi^h),$$

where we recall that $\chi^l = (1+t^l)/(1-\tilde{t}^h)$. We observe that country l's welfare cannot be expressed as a function only of χ^l and χ^h.

We now express the Nash first-order conditions for country l's optimal import and export tariffs as follows:

$$\begin{aligned} \frac{dU^l}{dt^l} &= \frac{f(\chi^l,\chi^h)}{1-\tilde{t}^h} + \frac{t^l}{1-\tilde{t}^h}\frac{df(\chi^l,\chi^h)}{dt^l} + \frac{\tilde{t}^l}{1-\tilde{t}^l}\frac{df(\chi^h,\chi^l)}{dt^l} + \frac{dCS(\chi^l,\chi^h)}{dt^l} = 0 \\ \frac{dU^l}{d\tilde{t}^l} &= \frac{t^l}{1-\tilde{t}^h}\frac{df(\chi^l,\chi^h)}{d\tilde{t}^l} + \frac{f(\chi^h,\chi^l)}{(1-\tilde{t}^l)^2} + \frac{\tilde{t}^l}{1-\tilde{t}^l}\frac{df(\chi^h,\chi^l)}{d\tilde{t}^l} + \frac{dCS(\chi^l,\chi^h)}{d\tilde{t}^l} = 0. \end{aligned}$$

We can re-write these first-order conditions as

$$\frac{dU^l}{dt^l} = \frac{f(\chi^l,\chi^h)}{1-\tilde{t}^h} + \left(\frac{t^l}{1-\tilde{t}^h} f_1(\chi^l,\chi^h) + \frac{\tilde{t}^l}{1-\tilde{t}^l} f_2(\chi^h,\chi^l) + CS_1(\chi^l,\chi^h)\right)\frac{\partial \chi^l}{\partial t^l} = 0$$

$$\frac{dU^l}{d\tilde{t}^l} = \frac{f(\chi^h,\chi^l)}{(1-\tilde{t}^l)^2} + \left(\frac{t^l}{1-\tilde{t}^h} f_2(\chi^l,\chi^h) + \frac{\tilde{t}^l}{1-\tilde{t}^l} f_1(\chi^h,\chi^l) + CS_2(\chi^l,\chi^h)\right)\frac{\partial \chi^h}{\partial \tilde{t}^l} = 0.$$

Using $\frac{\partial \chi^l}{\partial t^l} = \frac{1}{1-\tilde{t}^h} > 0$ and $\frac{\partial \chi^h}{\partial \tilde{t}^l} = \frac{1+t^h}{(1-\tilde{t}^l)^2} > 0$ under our assumptions, we may add the Nash first-order conditions, re-arrange terms and find the following necessary condition for the Nash equilibrium:

$$\begin{aligned} 0 &= f(\chi^l,\chi^h) + \frac{f(\chi^h,\chi^l)}{1+t^h} + \frac{t^l}{1-\tilde{t}^h}(f_1(\chi^l,\chi^h) + f_2(\chi^l,\chi^h)) \\ &\quad + \frac{\tilde{t}^l}{1-\tilde{t}^l}(f_2(\chi^h,\chi^l) + f_1(\chi^h,\chi^l)) + CS_1(\chi^l,\chi^h) + CS_2(\chi^l,\chi^h). \end{aligned}$$

At the symmetric Nash equilibrium, the necessary condition takes the following form

$$0 = \frac{f(\chi^N,\chi^N)}{1+t^N} + f(\chi^N,\chi^N) + (\chi^N - 1)\frac{df(\chi,\chi)}{d\chi}|_{\chi=\chi^N} + \frac{dCS(\chi,\chi)}{d\chi}|_{\chi=\chi^N}.$$

Recall now from (88) in the proof of Proposition 8 that we may express the first-order condition for efficient χ-symmetric tariffs as follows:

$$\frac{dU(\chi,\chi)}{d\chi} = 2[(\chi-1)\frac{df(\chi,\chi)}{d\chi} + f(\chi,\chi) + \frac{dCS(\chi,\chi)}{d\chi}] = 0.$$

This first-order condition determines χ^*. It is now direct to see that

$$\frac{dU(\chi,\chi)}{d\chi}|_{\chi=\chi^N} = -2\frac{f(\chi^N,\chi^N)}{1+t^N} < 0.$$

Thus, the symmetric Nash equilibrium is inefficient. Moreover, if the joint welfare function $U(\chi,\chi)$ is quasi-concave in χ, then $\chi^N > \chi^*$. Hence, starting at the symmetric Nash equilibrium, joint welfare is increased by any combination of tariff changes that results in a symmetric reduction in $\chi^H = \chi^F$. Both countries thus gain if they exchange small and symmetric changes in tariffs that reduce $\chi^H = \chi^F$.

8 References

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