Entry and Welfare in General Equilibrium with Heterogeneous Firms and Endogenous Markups*

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Abstract

We consider the efficiency of market entry in single- and two-sector closed-economy versions of the Melitz-Ottavanio (MO) model, where differently from the MO model our two-sector model does not involve an outside good. For each model version, we assess whether the market level of entry is efficient relative to the second-best setting in which the planner can control only the level of entry. Focusing on entry levels that induce selection, we show that the market level of entry is efficient in the single-sector model. For a two-sector MO model without an outside good, we show that the welfare results are exactly similar to those in the one-sector model when the two sectors are symmetric. When the two sectors are asymmetric and the level of asymmetry is sufficiently small, we identify a perturbation indicating a sense in which the market level of entry into the "high-demand" sector is excessive. This inefficiency arises due to the presence of an additional aggregator in MO preferences.

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1 Introduction

We consider the efficiency of market entry in single- and two-sector closed-economy versions of the Melitz and Ottavanio (2008) model, where differently from the Melitz-Ottaviano (MO) model our two-sector model does not involve an outside good. For each model version, we assess whether the market level of entry is efficient relative to the second-best setting in which the planner can control only the level of entry. When an inefficiency is identified, we also characterize welfare-improving adjustments in entry levels.

The MO model posits quasi-linear preferences and has two sectors, where one sector is a differentiated sector and the other sector is an outside-good sector that produces a homogeneous good under perfect competition.[1] Bagwell and Lee (2020) examine the efficiency of market entry in the closed-economy MO model, finding that entry into the differentiated sector is excessive (inadequate) (efficient) if and only if $\alpha > 2 \cdot c_D^m$ ($\alpha < 2 \cdot c_D^m$) ($\alpha = 2 \cdot c_D^m$), where α is a demand parameter with higher values indicating a greater preference for differentiated goods relative to the outside good and where c_D^m is the cutoff cost level for surviving varieties as determined in the market equilibrium.[2] The cutoff level c_D^m is independent of α in this outside-good model. As Bagwell and Lee discuss, an understanding of the efficiency properties of market entry is essential for understanding trade policy and agreements in the two-country MO model of trade.

We focus here on the efficiency of market entry in a closed-economy setting for formulations of the MO model in which the outside good is absent. We thus evaluate the efficiency of entry under monopolistic competition and heterogeneous firms while allowing for variable markups and general-equilibrium income effects. As we discuss in more detail below, the differentiated sector in the MO model is also attractive for analysis, since the associated preferences are tractable and fail to exhibit even "generalized additive separability" (Bertoletti and Etro, 2022; Pollak, 1972) due to the existence of multiple aggregators.

We begin by examining a single-sector version of the MO model. Our first main result is that the market level of entry is efficient among entry levels that induce at least some selection. This result obtains even though entry has external effects on firms and consumers. The result also carries the following specific implication: the entry inefficiency characterized by Bagwell and Lee (2020) is attributable to the fact that the original MO model has multiple sectors.

[1] Specifically, in the MO model, the outside good enters utility in a linear and additive way, implying that income effects can be ignored. See also Ottaviano, Tabuchi, and Thisse (2002) for a treatment in a regional context.

[2] Bagwell and Lee (2020) primarily analyze trade policy under a two-country setup. In their Section 3.3, they explore a closed-economy version to clarify the underlying mechanism.

We next examine whether the inefficiency of market entry in the MO model is sensitive to the way in which the second sector is modeled. To this end, we replace the assumption that the second sector is an outside-good sector with the alternative assumption that the second sector is another differentiated sector, where the upper-tier consumer utility function is additively separable across the two sectors. We conduct two exercises. In the first exercise, we consider a symmetric setting in which the demand parameter α takes the same value in both sectors: $\alpha_1 = \alpha_2$. For this symmetric setting, we analyze the implications of a small perturbation in which the planner symmetrically changes the levels of entry in the two sectors. Our second main result is that, for this symmetric setting and relative to this perturbation class, the market level of entry satisfies the first-order conditions for welfare maximization, just as in the one-sector model. Our second exercise allows that the sectors may be asymmetric in that $\alpha_1 \neq \alpha_2$ is allowed. For this setting, we consider a small perturbation in which the planner increases the level of entry into sector 1 while simultaneously decreasing the level of entry into sector 2 in such a manner as to ensure that the marginal utility of income λ for the consumer is unaltered.[3] As our third main result, we show that, for sufficiently small asymmetries, this perturbation raises (lowers) (does not change) welfare if and only if $\alpha_1 < \alpha_2$ $(\alpha_1 > \alpha_2)$ $(\alpha_1 = \alpha_2)$. Thus, in this sense, the market provides excessive entry into the sector $s \in \{1, 2\}$ with the highest value for α_s.

To interpret our findings, we begin with the single-sector model. Additional entry in this model introduces a tradeoff via the resource constraint, since additional entry also impacts variety-level consumption through induced changes in the marginal utility of income λ and the critical cost cutoff level c_D. Focusing on entry levels that induce some selection, we find that the market trades off these considerations in an efficient manner. We show that this finding can be understood by considering the impact of additional entry on aggregate output.[4] Starting at the market equilibrium, additional entry introduces offsetting effects on λ and c_D such that the number of surviving varieties (the extensive margin) and the expected variety-level output conditional on survival (the intensive margin) are each unaffected to the first order, ensuring that aggregate output and thus welfare are also unaffected to the first order.[5]

For the two-sector model, we show that our first exercise may be interpreted in a manner that is analogous to the interpretation just given for the one-sector model. Just as in our analysis of the one-sector model, the symmetric change in entry levels induces

[3]Formally, λ is the Lagrange multiplier for the budget constraint in the consumer optimization problem.

[4]As Demidova (2017) shows, welfare can be expressed as a function of aggregate output in the one-sector MO model.

[5]Thus, even though the market levels for λ and c_D are not first best (as Bagwell and Lee, 2023 show), the planner cannot advantageously manipulate these distortions by altering the level of entry.

changes in λ and the symmetric critical cost cutoff levels, which offset each other. Indeed, when the two-sector model has a symmetric setting and is subjected to a symmetric change in sectoral entry levels, the results are exactly similar to those in the one-sector model.

Our second exercise for the two-sector model, however, introduces an additional consideration, since in the asymmetric two-sector model the market may misallocate resources *across* sectors. To isolate this consideration, we start at the market equilibrium and increase the level of entry into the first sector while adjusting the level of entry into the second sector so as to ensure that the marginal utility of income λ is unchanged. We show that a small perturbation of this kind necessarily involves a reduction in the level of entry into the second sector. With this experiment, we thus eliminate intensive margin effects that are induced via a change in λ. Assuming that the level of asymmetry is sufficiently small, we show that this perturbation lowers welfare when $\alpha_1 > \alpha_2$, a finding that in this sense is consistent with a business-stealing intuition under which the market provides excessive entry into "high-demand" sectors.

Our second exercise shares qualitative features with the closed-economy analysis by Bagwell and Lee (2020); in both cases, the marginal utility of income λ remains fixed, and so variety-level consumption is not impacted by changes in λ. In addition, we find that the market provides excessive entry into the sector s with the highest value for α_s, a finding which is broadly analogous to the findings by Bagwell and Lee regarding excessive entry into the differentiated sector in the model with an outside good when α is high (namely, when $\alpha > 2 \cdot c_D^m$ in that model).[6]

We do not intend to argue against the value of partial-equilibrium models with an outside-good sector and quasi-linear preferences. Such models are highly tractable and provide valuable insights for a range of policy analyses. At the same time, a general-equilibrium model is appropriate for analyses seeking to include the income effects of policies. Two-sector models with an imperfectly competitive sector and an outside-good sector are also typically structured in such a way as to impose intersectoral markup heterogeneity: markups are positive in the imperfectly competitive sector and absent in the (competitive) outside-good sector. This built-in asymmetry can have implications for resource misallocation.[7]

By comparison, in the two-sector model considered in the current paper, the average markup is symmetric across sectors, even when preferences are asymmetric ($\alpha_1 \neq \alpha_2$).[8]

[6] At a broad level, our second exercise thus suggests possible directions in which the trade-policy findings of Bagwell and Lee may extend to a multi-sector MO model without an outside good. A complete analysis of this relationship, however, would require placing the multi-sector model considered here into a two-country model with trade policies. This is beyond the scope of the current paper.

[7] See Lerner (1934) for an early discussion.

[8] We define the average markup in a sector as the ratio of the average price to the average marginal

In this way, we shut down the possibility that resources are misallocated due to markup asymmetry across sectors. Since both sectors are imperfectly competitive, our approach also differs in that a reallocation of entry across sectors creates subtle externalities for consumer and producer interests in both sectors. These externalities account for the welfare gain from entry reallocation that we establish for the two-sector model with asymmetric preferences.

To provide a deeper understanding of these externalities, we examine the role of the demand parameter η. This parameter multiplies the aggregate quantity, which enters negatively in the utility function, resulting in non-additively separable preferences and a demand function with two aggregators, λ and the aggregate quantity. As aggregate consumption increases, the η parameter causes the marginal utility of each individual variety to decrease, an effect we refer to as the aggregate-consumption (or η) penalty. In a multi-sector model, the free entry condition requires that if positive entrants exist in both sectors, they should face the same level of expected profits. We conjecture that the free entry condition imposes an excessive aggregate-consumption penalty on entrants in high-demand sector. To confirm this hypothesis, we decompose the overall welfare impact for the second exercise into the impact on the η term and the impact on the remaining terms. We find that the first-order impact on the η term aligns with the overall impact, while the other terms exhibit the opposite sign. Based on the analysis, we thus establish a sense in which the η penalty underlies excessive entry in the high-demand sector.

This aggregate-consumption penalty channel, while intuitively straightforward, does not arise under additively separable preferences. Without this penalty, there is no mechanism to equalize the expected profits across two asymmetric sectors. We examine entry efficiency as in the second exercise when $\eta = 0$ (i.e. quadratic preferences) and find that the free entry condition then causes all firms to enter the high-α sector, resulting in a corner solution. In the online appendix, we also explore alternative additive preferences and examine a similar entry efficiency problem when two CES subutilities are additively combined at the upper-tier. We again reach a corner solution in the market equilibrium.

Behrens, Mion, Murata and Suedekum (2020) also investigate misallocation in a multi-sector model with heterogeneous firms and monopolistic competition. In their model, the lower tier is an additively separable utility function whereas the upper tier utility function is in a general form. They compare the first-best allocation to the market solution and find that the efficiency of resource allocation depends on the elasticities of both the upper-tier utility function and the lower-tier utility function. The distortions in one sector depend

cost in that sector, where averages are taken over surviving firms. The average markup is equal across sectors in our two-sector model, even after entry is reallocated away from the market equilibrium level. We note as well that given free entry, the difference between the average price and average marginal cost in a sector is also independent of the sector at the market equilibrium. This difference, however, may become asymmetric across sectors following a reallocation of entry.

on the characteristics of all sectors, which shares a similar insight with our second exercise in that excessive entry in the high-α sector is accompanied by insufficient entry in the low-α sector. Our analysis differs from Behrens et al. in that we analyze the second-best allocation and consider a different utility function where the upper tier exhibits perfect substitutability and the lower tier is not additively separable. This different specification allows us to focus on the role of the aggregate-consumption penalty.[9]

We also investigate the possible role of an additional demand aggregator by examining a firm's entry decision in a single-sector model when $\eta = 0$. In this case, our demand function has a single aggregator λ and the associated preferences belong to the family of generalized additively separable (GAS) preferences. We consider the market-determined entry level when $\eta = 0$ and find the same entry level as when $\eta > 0$. When $\eta = 0$, the Lagrange multiplier λ operates similarly to the aggregate-consumption penalty, as a larger aggregate consumption level lowers the consumption of each individual variety by adjusting the budget constraint.

Fally (2022) argues for greater flexibility by incorporating an additional aggregator in a single-sector model with GAS preferences. Our findings illustrate the value of this increased flexibility. In our single-sector model, the inclusion of an additional aggregator via the η penalty does not affect firms' entry decisions, since as noted the Lagrange multiplier λ already fulfills a comparable role by diminishing expected profit with additional entry. However, within our multi-sector model, the incorporation of an additional aggregator becomes necessary to generate sector-specific influences stemming from a firm's entry. This alteration is responsible for the transformation of firms' market entry levels from corner solutions to interior solutions in our multi-sector model.

We also examine the role of our distributional assumption for our finding of efficient entry in the second-best setting for the single-sector MO model. As in Melitz and Ottaviano (2008), we assume that costs follow a Pareto distribution with a range of $[0, c_M]$ where $c_M > 0$. The associated distribution of productivities is thus unbounded above. Bagwell and Lee (2023) analyze the first-best allocation in the single-sector MO model and find that the market delivers the first-best level of entry. Interestingly, under this dis-

[9] If we assume some degree of complementarity in the upper-tier utility function, this preference for variety would allocate some resources to every sector (i.e. no corner solution). Hence, it would not be trivial to separate the role of the aggregate-consumption penalty from that of the complementarity in the upper-tier decision in the intersectoral resource allocation. Segerstrom and Sugita (2015) also explore a multi-sector version of CES model with monopolistic competition and firm heterogeneity, similar to ours in the online appendix. They show that trade liberalization in a sector more strongly increases the average productivity of non-liberalized sectors compared to the liberalizing sector, which cannot be explained by a standard single-sector model including Melitz (2003). While they assume a general upper-tier utility function, they only consider the interior-solution case at the upper-tier decision. Bagwell and Lee (2018) analyze a multi-sector model comprised of a CES differentiated sector and an outside-good sector, where the upper-tier utility function takes a quasi-linear form. They also focus on an interior solution and show that the market provides insufficient entry in the differentiated sector.

tributional assumption, the first-best level of entry is the same as the second-best level of entry found in the current paper.[10] As Bagwell and Lee (2023) note, Bertoletti, Etro, and Simonovska (2018) and Bertoletti and Etro (2021) explore different preference structures but still discover, under an unbounded Pareto distribution, that the market entry level is efficient and takes the same form. For their first-best analysis, Bagwell and Lee show further that the finding of efficient entry fails to hold under the alternative assumption of a bounded Pareto distribution that takes the simple form of a uniform distribution for costs over the interval $[c_L, c_U]$ with $c_U > c_L > 0$. In the current paper, we find that the second-best efficiency of the market entry level also depends on the assumption of an unbounded Pareto distribution. Although we are unable to derive a closed-form solution for the second-best entry level when $c_L > 0$, our numerical analysis indicates that the market exhibits excessive entry, consistent with Bagwell and Lee's findings regarding the first-best entry level.

In other related research, Demidova (2017) studies optimal unilateral tariffs in a two-country model that utilizes the single-sector version of the MO model. By contrast, we focus here on the efficiency properties of the market level of entry. Demidova notes that the level of entry in the single-sector model is in fact indepenent of tariffs and trade costs. We allow the planner to choose directly the level of entry, where the resulting market outcomes can be replicated with the appropriate selection of entry tax/subsidy policies. We thus consider different policy instruments than does Demidova.

Our work is also related to an Industrial Organization literature that takes a partial-equilibrium perspective and considers the efficiency of entry in an imperfectly competitive sector when firms are symmetric, an outside-good sector exists and preferences are quasi-linear. Prominent contributions to this literature include Mankiw and Whinston (1986) and Spence (1976). For the second-best problem of a planner who can control only the number of firms, this literature finds that the level of entry is typically inefficient, due to the associated business-stealing and consumer-surplus externalities. Differently from this research, we eliminate the outside-good sector, include heterogeneous firms, and establish a second-best efficiency result for the market level of entry in the one-sector model.[11]

Campolmi et al (2014) consider a two-sector monopolistic competition model, where CES preferences and symmetric firms are specified for the differentiated sector, the other sector is an outside-good sector and the upper-tier utility function takes a Cobb-Douglas form. They find that the market level of entry is inefficient and too low, and they show that a wage subsidy that targets the monopolistic distortion can implement the first-best outcome. Bagwell and Lee (2018) examine the efficiency of entry in a two-sector model of

[10] Demidova (2017) and Simonovska (2015) report the same market entry level in similar single-sector models of international trade. However, their studies do not investigate first- or second-best solutions.

[11] Other important related work with symmetric firms includes Bertoletti and Etro (2016), Dixit and Stiglitz (1977), Fally (2022), Matsuyama and Ushchev (2020) and Parenti, Ushchev and Thisse (2017).

monopolistic competition, where CES preferences and heterogeneous firms are specified for the differentiated sector, the other sector is an outside-good sector and the upper-tier utility function takes a quasi-linear form. They also find that the market level of entry is inefficient and too low. Like Bagwell and Lee (2020), Nocco et al (2014) consider the efficiency properties of the original MO model with an outside good. Nocco et al also characterize first-best outcomes and associated implementation policies.[12]

Our analysis of the two-sector model also relates to work by Epifani and Gancia (2011). They examine a multi-sector model that features between- but not within-sector heterogeneity. For a class of models, they show that, under free entry and when the preference for variety differs across sectors, there exists no markup distribution such that the market equilibrium replicates the first-best allocation. Markup symmetry is thus not sufficient for first-best efficiency in this setting. Similarly, in our analysis of the two-sector model with asymmetric preferences ($\alpha_1 \neq \alpha_2$) across sectors, we establish a welfare gain from a reallocation of entry across sectors even though the average markup does not differ across sectors. But our analysis also differs in several respects. We include within-sector firm heterogeneity, establish a second-best efficiency result for the market level of entry in the single-sector and symmetric two-sector models, and construct a specific welfare-improving entry reallocation in our asymmetric two-sector model that entails reducing entry into the "high-demand" sector.

Finally, our work is related to research on first-best outcomes in single-sector models. Dhingra and Morrow (2019) examine a family of single-sector monopolistic competition models featuring heterogeneous firms and additively separable preferences. For this family, they show that the market outcome is first best if and only if preferences take the CES form. By contrast, the preferences that we explore do not fit in this family, and we also restrict attention to second-best intervention that targets only the number of entrants. Bagwell and Lee (2023) characterize the first-best allocation for the single-sector MO model considered here. In comparison to the first-best optimum, the market provides the same level of entry but too little selection; thus, the market provides too many varieties and allocates too little (much) production to low (high) cost realizations. Bertoletti et al. (2018) and Bertoletti and Etro (2021) consider single-sector models while allowing for other preferences. Bertoletti et al. explore indirectly additive preferences, and Bertoletti and Etro examine a broader set of symmetric GAS preferences. The defining feature of GAS preferences is the existence of a single aggregator of prices or quantities in the demand system. This feature distinguishes GAS preferences from the MO preference, which has two aggregators due to the existence of the η term.

The paper is organized as follows. In Section 2, we develop the one-sector MO model

[12]See also Spearot (2016) for a multi-sector, multi-country amended version of the MO model in which the outside-good sector is removed. He provides counterfactual analyses of trade policies.

and present our welfare finding for this model. In Section 3, we present the two-sector MO model that we study while allowing for entry policies (i.e., subsidies or taxes for the cost of entry). We then analyze our two welfare exercises for the two-sector MO model in Section 4. In Section 5, we discuss extensions and robustness. In Section 6, we show the outcomes induced by the planner's direct choice of entry levels alternatively can be induced by an appropriate choice of entry tax/subsidy policies, and vice versa. Section 7 concludes. Remaining proofs are contained in the Appendix.

2 One-sector MO model

In this section, we analyze the one-sector MO model. Our analysis employs the following timeline:

1. A planner decides the number (mass) of entrants.

2. Entrants pay a fixed cost to observe their respective marginal costs and decide how much to produce, including whether to produce or not. Their respective decisions shape the prices and number of varieties available to consumers.

3. Consumers maximize utility under the given prices and varieties.

We solve the model by starting with the final stage and working backwards, in accordance with standard procedure.[13]

2.1 Consumer's problem

The economy consists of a unit mass of identical consumers, each supplying a unit of labor in inelastic manner to a competitive labor market. We normalize the wage as 1. Consumers own symmetric shares of any aggregate net profit, the value of which an individual consumer takes as fixed when choosing consumption.

The consumer's welfare maximization problem can be written as follows

$$\max_{\{q_i\}_{i\in\Omega}} U = \alpha \int_{i\in\Omega} q_i di - \frac{1}{2}\gamma \int_{i\in\Omega} (q_i)^2 \, di - \frac{1}{2}\eta \left(\int_{i\in\Omega} q_i di \right)^2 \tag{1}$$

s.t.

$$\int_{i\in\Omega} p_i q_i di = 1 + \Pi \tag{2}$$

[13] We investigate market efficiency in comparison to the solution of this second-best problem. A related analysis is presented by Bagwell and Lee (2023) who explore market efficiency in relation to the first-best problem in which the planner determines the entry and output decisions for each cost type.

where q_i and p_i represent the consumption and price of variety i in the set Ω of available varieties, wage income is normalized as 1 and Π refers to the aggregate net profit. We assume that the preference parameters α, γ and η are all positive.

To solve the consumer's problem, we construct the Lagrangian

$$L = U + \lambda \left(1 + \Pi - \left(\int_{i \in \Omega} p_i q_i di\right)\right),$$

where $\lambda \geq 0$ is the multiplier for the consumer's optimization problem above. Letting $Q \equiv \int_{i \in \Omega} q_i di$, we represent the first-order condition with respect to q_i as

$$\alpha - \gamma \cdot q_i - \eta \cdot Q = \lambda p_i. \tag{3}$$

Integrating (3) over the set of varieties for which $q_i > 0$ and letting N be the measure of consumed varieties in Ω, we obtain

$$\frac{\alpha - \lambda \bar{p}}{\eta + \frac{\gamma}{N}} = Q,$$

where $\bar{p}$ is the average price of consumed varieties.

Assuming $\lambda > 0$, we may further characterize consumer demand. Using (3), we see that $p_i = \frac{(\alpha - \eta \cdot Q - \gamma \cdot q_i)}{\lambda}$ for consumed varieties. Let us now define $p^{\max}$ as the "choke price." From the foregoing, we may confirm that

$$p^{\max} \equiv \frac{\alpha - \eta \cdot Q}{\lambda} = \frac{1}{\lambda}\left(\frac{\gamma \cdot \alpha + \lambda \cdot \eta \cdot N \cdot \bar{p}}{\eta \cdot N + \gamma}\right). \tag{4}$$

Using (3) and (4), the inverse demand can be written as

$$p^d(q) = p^{\max} - \frac{\gamma}{\lambda} \cdot q. \tag{5}$$

2.2 Firm's problem

Profit maximization for a firm with marginal production cost c delivers the profit function

$$\pi(c) = \max_q \left(p^d(q) - c\right) q.$$

For this model of monopolistic competition, the firm takes Q, λ and consequently the demand intercept $p^{\max}$ as given when determining its profit-maximizing output. Using

(5), we characterize the solution to the firm's problem as

$$q(c) = \frac{\lambda (p^{\max} - c)}{2\gamma}. \tag{6}$$

This solution determines the profit-maximizing price and maximized profit of the firm:

$$p(c) = \frac{p^{\max} + c}{2} \tag{7}$$

$$\pi(c) = \frac{\lambda}{4\gamma} (p^{\max} - c)^2 .$$

The firm produces a positive quantity of its variety provided that its cost realization is not higher than the demand intercept. In other words, a Zero Cutoff Profit (ZCP) condition determines the cost cutoff c_D as $\pi(c_D) = 0$ or equivalently

$$p^{\max} = p(c_D) = c_D, \tag{8}$$

where we assume that $c_D > 0$.

Following Melitz and Ottaviano (2008), we assume that costs follow a Pareto distribution

$$G(c) = \left(\frac{c}{c_M}\right)^k \tag{9}$$

for $c \in [0, c_M]$ where $k > 1$ and $c_M > 0$. We also impose an implicit restriction on our parameters and assume that $c_D < c_M$, to ensure that some selection occurs. We consider this as the most interesting case for the heterogeneous-firms model. We return to this assumption as well as our positive value assumptions ($\lambda > 0$, $c_D > 0$) below.

Given this distribution, we have that

$$\overline{c} \equiv E(c|c \leq c_D) = \left(\frac{k}{k+1}\right) c_D.$$

Using (7), (9), and $p^{\max} = c_D$, we find that the average price can be represented as

$$\overline{p} \equiv E(p(c)|c \leq c_D) = \left(\frac{c_D + \overline{c}}{2}\right) = \left(\frac{2k+1}{2(k+1)}\right) c_D.$$

Observe that the average markup, $\overline{\mu} \equiv \overline{p}/\overline{c}$, is a simple function of the parameter k:

$$\overline{\mu} = \frac{2k+1}{2k}.$$

Referring to (4) and using $p^{\max} = c_D$ and the expression just derived for $\bar{p}$, we can represent the number of varieties as

$$N = \frac{\gamma\left(\alpha - \lambda \cdot p^{\max}\right)}{\lambda \cdot \eta \cdot \left(p^{\max} - \bar{p}\right)} = \frac{\gamma\left(\alpha - \lambda \cdot c_D\right)}{\lambda \cdot \eta\left(c_D - \bar{p}\right)} = \frac{2\left(k+1\right)\gamma}{\eta}\frac{\left(\alpha - \lambda \cdot c_D\right)}{\lambda \cdot c_D}. \tag{10}$$

From the last expression in (10), we see that, once values for c_D and λ are obtained, the value for the number of available varieties is determined. We note that N is strictly decreasing with respect to c_D for a given value of λ.

The number of available varieties can also be expressed as a function of the number of entrants, N_E, and the cost cutoff level as $N = N_E \cdot G(c_D)$. Hence, the value for the number of available varieties can also be determined given the number of entrants and the cost cutoff level.

Finally, using (10), the Pareto distribution and $N = N_E \cdot G(c_D)$, we can express the relation between N and N_E as:

$$N_E = \frac{2\left(k+1\right)\gamma\left(c_M\right)^k}{\eta}\frac{\left(\alpha - \lambda \cdot c_D\right)}{\lambda \cdot \left(c_D\right)^{k+1}}. \tag{11}$$

The expression in (11) will be an important ingredient in our analysis below when we explore the implications of different values for N_E for λ, c_D and consumer welfare.

The expression in (11) represents the demand for entering firms in the economy, which decreases with c_D and λ. To demonstrate that N_E decreases with c_D, suppose c_D rises while holding λ constant. In this case, the survival rate $G(c_D)$ increases, and equation (10) implies that N must decrease. Since $N = N_E \cdot G(c_D)$, a lower N with a higher $G(c_D)$ necessitates a reduction in N_E. The demand for N_E decreases with λ since a higher value for λ raises the overall quantity allocation under given c_D as shown by (6) and (8). Intuitively, the role of λ is to adjust the consumer's budget constraint. Hence, a higher value for λ implies more resource allocation to intensive margin consumption $q\left(c\right)$ and lower resource allocation (i.e. demand) to N_E. To determine the equilibrium value of N_E, an additional condition on its supply is required. In the market equilibrium, this condition is governed by the Free Entry condition, while in our second-best setting, it is determined by the planner's decision. We provide these conditions in the following Section 2.3.

2.3 Planner's problem

We are now ready to consider the planner's choice of N_E. The planner seeks to choose N_E so as to maximize consumer welfare under (i) a resource constraint derived from (2), (ii) a constraint on the relationship between N_E, c_D and λ as given in (11), and (iii) a profit-maximizing constraint under which the quantity of variety i consumed is determined by the corresponding firm's cost realization and profit-maximizing output (including zero), as implied by (6) and $p^{\max} = c_D$. To state the planner's problem, we proceed by showing that the objective and constraints can be written in terms of α, N_E, c_D and λ.[14]

We start with the objective function U defined in (1). To simplify U, we use (6), (11), $p^{\max} = c_D$ and the Pareto distribution to show that we may rewrite each term in (1) in terms of c_D and λ.

This rewriting is accomplished through the establishment of two claims. The first claim is that

$$\int_{i\in\Omega} q_i di = N_E \int_0^{c_D} q\left(c\right) dG\left(c\right) = \frac{1}{\eta}\left(\alpha - \lambda \cdot c_D\right) \tag{12}$$

The first equality in (12) follows from profit-maximizing behavior. As we show in the Appendix, the second equality can be confirmed using (6), $p^{\max} = c_D$, (11) and the Pareto distribution. With (12) established, we can rewrite the first and third terms in (1) in terms of c_D and λ. The second claim is that

$$\int_{i\in\Omega} \left(q_i\right)^2 di = N_E \int_0^{c_D} q\left(c\right)^2 dG\left(c\right) = \frac{1}{\eta}\frac{\left(\alpha - \lambda \cdot c_D\right)\lambda \cdot c_D}{\gamma\left(2+k\right)}. \tag{13}$$

The first equality in (13) also follows from profit-maximizing behavior. In the Appendix, we show that the second equality can be confirmed using (6), (11), $p^{\max} = c_D$ and the Pareto distribution. With (13) established, we can rewrite the second term in (1) in terms of c_D and λ.

With the two claims established, we now plug (12) and (13) into (1). After simplification, we obtain that

$$U = \frac{\left(\alpha - \lambda \cdot c_D\right)}{2\eta}\left(\alpha + \left(\frac{1+k}{2+k}\right)\lambda \cdot c_D\right).$$

We may thus rewrite U as a function of (α, c_D, λ). Formally, we write

$$U = u\left(\alpha, c_D, \lambda\right)$$

where

$$u\left(\alpha, c_D, \lambda\right) \equiv \frac{\left(\alpha - \lambda \cdot c_D\right)}{2\eta}\left(\alpha + \left(\frac{1+k}{2+k}\right)\lambda \cdot c_D\right). \tag{14}$$

We turn next to the resource constraint as given by (2). In order to simplify the

[14] For later use in a multi-sector setup, we include α as an independent variable for our objective and constraint functions as defined below.

planner's problem, we rewrite (2) under utility- and profit-maximizing behavior as

$$N_E \int_0^{c_D} p(c)\, q(c)\, dG(c) = 1 + N_E \left[\int_0^{c_D} (p(c) - c)\, q(c)\, dG(c) - f_E \right],$$

where the bracketed term on the RHS of this equation represents the expected profit for a firm that pays the fixed cost $f_E > 0$ to observe its cost realization. After simplification, the resource constraint takes the following form:

$$N_E \left(\int_0^{c_D} c \cdot q(c)\, dG(c) + f_E \right) = 1. \tag{15}$$

Using (6), $p^{\max} = c_D$, (11) and the Pareto distribution, and after simplification, we can write the resource constraint(15) as

$$R(\alpha, c_D, \lambda) = \frac{\eta(2+k)}{\gamma \cdot \phi} \tag{16}$$

where

$$R(\alpha, c_D, \lambda) \equiv \frac{(\alpha - \lambda \cdot c_D)}{\lambda \cdot (c_D)^{k+1}} \left(\frac{(c_D)^{k+2} k \cdot \lambda}{\gamma \cdot \phi} + 1 \right) \tag{17}$$

and $\phi = 2(k+1)(k+2)(c_M)^k f_E$.

Next, using (11), we also define the number of entrants N_E as a function of (α, c_D, λ):

$$N_E = N_e(\alpha, c_D, \lambda) \equiv \frac{2(k+1)\gamma (c_M)^k}{\eta} \frac{(\alpha - \lambda \cdot c_D)}{\lambda \cdot (c_D)^{k+1}}. \tag{18}$$

Finally, we have already embedded the profit-maximizing constraints into our representations of the utility function, the resource constraint and the constraint on the relationship between N_E, c_D and λ as given as given by (14), (16), (17) and (18), respectively.

In this second-best setting, we model the planner as choosing a value N_E for the number of entrants, with c_D and λ then determined by the market in accordance with the following system:

$$R(\alpha, c_D, \lambda) = \frac{\eta(2+k)}{\gamma \cdot \phi} \tag{19}$$

and

$$N_E = N_e(\alpha, c_D, \lambda) > 0 \tag{20}$$

where $R(\alpha, c_D, \lambda)$ and $N_e(\alpha, c_D, \lambda)$ are defined in (17) and (18), respectively.

Before stating the planner's problem and characterizing its solution, we require conditions under which our positive-value assumptions ($\lambda > 0$, $c_D > 0$) and our restriction that $c_D < c_M$ are satisfied. Drawing on arguments developed in a different context by

Bagwell and Lee (2023), we may easily confirm for $N_E \in (0, 1/f_E)$ that there exists a unique solution to (19) and (20) that satisfies $\lambda > 0$ and $c_D > 0$; furthermore, for a given $c_M > 0$, there exists $\widetilde{N}_E \in (0, 1/f_E)$ such that the solution satisfies $c_D < c_M$ if and only if $N_E > \widetilde{N}_E$.[15] We can also show that $\widetilde{N}_E$ goes to zero as c_M or α approaches infinity.[16] Thus, our focus on the case where some selection occurs can be understood as corresponding to a lower bound on entry, where this lower bound can be made arbitrarily small with further parameter restrictions.

Focusing on a planner who chooses over entry values that induce selection with $c_D < c_M$, we now represent *the planner's problem* as

$$\max_{N_E \in (\widetilde{N}_E, 1/f_E)} u(\alpha, c_D, \lambda) \text{ s.t. (19) and (20),}$$

where $u(\alpha, c_D, \lambda)$is defined in (14).

To understand the planner's problem, suppose that the planner entertains a specific value for $N_E \in (\widetilde{N}_E, 1/f_E)$. Given this value, we may regard the constraints (19) and (20) as defining a 2×2 system of equations, in which c_D and λ are endogenous while N_E is exogenous. Accordingly, we can conduct a traditional comparative statics exercise to determine how c_D and λ vary with respect to N_E. We can then feed this information into the planner's optimization problem with respect to N_E. Finally, while our representation of the planner's problem does not explicitly include the number of available varieties, N, we recall from (10) that this value can be easily recovered once c_D and λ are determined.

For the planner's problem, our main goal is to determine whether, starting at market equilibrium, additional entry raises welfare or not. To this end, we first define the market equilibrium solution $(c_D^{mkt}, \lambda^{mkt}, N_E^{mkt})$ as the solution to the 3×3 system of equations in which c_D, λ and N_E are endogenous variables and the three equations are (19), (20) and a third equation, the Free Entry condition, which is defined as follows:

$$\int_0^{c_D^{mkt}} (p(c) - c)\, q(c)\, dG(c) = f_E. \tag{21}$$

For our purposes here, the key property of the market solution is that a relationship

[15] $N_E \leq 1/f_E$ is necessary for feasibility. This can be easily seen from the resource constraint (15).

[16] Bagwell and Lee (2023) analyze a first-best setting in which the planner rather than the market chooses the quantity function. To this end, they examine a sub-problem in which the number of entrants is fixed. In their Appendix C, they characterize the solution to this sub-problem when selection occurs. Following steps employed there, we may establish the claims made in this paragraph. These claims hold in both settings even though the respective values taken by c_D, λ and $\widetilde{N}_E$ differ across the two settings.

between c_D^{mkt} and λ^{mkt} is implied:

$$\lambda^{mkt} = \frac{\gamma \cdot \phi}{\left(c_D^{mkt}\right)^{k+2}}. \tag{22}$$

This relationship follows from (21), after using (6), (7), setting $p^{\max} = c_D$ and using the Pareto distribution.

To pin down N_E^{mkt}, we solve (22) for $\left(c_D^{mkt}\right)^{k+2} \lambda^{mkt}$ and plug the solution into the (19). We then solve the resulting expression in (19) for $\left(\alpha - \lambda^{mkt} c_D^{mkt}\right) / \lambda^{mkt} \left(c_D^{mkt}\right)^{k+1}$ and plug the solution into (20). With these steps, we obtain

$$N_E^{mkt} = \frac{1}{f_E\,(k+1)} \tag{23}$$

where (23) is identical to the one presented by Demidova (2017). In turn, we can pin down c_D^{mkt} and λ^{mkt} by using (19), (20) and (23).

Two issues still need to be addressed in our characterization of the market equilibrium solution. First, we must verify that (19), (20), and (23) admit a unique solution satisfying our positive-value assumptions ($\lambda^{mkt} > 0$, $c_D^{mkt} > 0$, $N_E^{mkt} > 0$). Second, we must make restrictions to ensure $c_D^{mkt} < c_M$. Bagwell and Lee (2023) address both issues in their Appendix E. They show that there exists a unique solution to a system equivalent to the one comprised of (19), (20) and (23) satisfying our positive-value assumptions. They also show that if $\frac{\eta(k+2)}{\alpha(k+1)} < c_M$ and $f_E > 0$ is sufficiently small, then $c_D^{mkt} < c_M$. We maintain these restrictions in our analysis. As Bagwell and Lee discuss, these restrictions also ensure that $N_E^{mkt} > \widetilde{N}_E$.

Let us now represent the solutions to the 2×2 system of constraints (19) and (20) as $c_D(N_E)$ and $\lambda(N_E)$. For $N_E \in (\widetilde{N}_E, 1/f_E)$, we may represent the first-order condition for the planner's problem as

$$\frac{du}{dN_E} = \frac{\partial u}{\partial c_D}\frac{dc_D}{dN_E} + \frac{\partial u}{\partial \lambda}\frac{d\lambda}{dN_E} = 0. \tag{24}$$

In the Appendix, we show that the first-order condition (24) is uniquely satisfied at the market solution:

$$\frac{du}{dN_E}|_{N_E = N_E^{mkt}} = \frac{\partial u}{\partial c_D}\frac{dc_D}{dN_E} + \frac{\partial u}{\partial \lambda}\frac{d\lambda}{dN_E}|_{N_E = N_E^{mkt}} = 0. \tag{25}$$

At the market solution, we find that $\frac{\partial u}{\partial c_D} < 0$, $\frac{\partial u}{\partial \lambda} < 0$, $\frac{dc_D}{dN_E} < 0$ and $\frac{d\lambda}{dN_E} > 0$, where the various terms balance out so as to satisfy (25). We also establish that the second-order condition holds at the market solution:

$$\frac{d^2u}{d(N_E)^2}|_{N_E=N_E^{mkt}} < 0. \tag{26}$$

Based on these findings, we conclude that the market solution uniquely maximizes the planner's welfare over all $N_E \in (\widetilde{N}_E, 1/f_E)$ and thus solves the planner's problem. The following proposition summarizes our first main result:

Proposition 1 *The entry level at the market equilibrium in the one-sector MO model uniquely solves the planner's problem.*

Proof. The proof of Proposition 1 is completed in the Appendix. ■

Proposition 1 is of particular interest relative to previous works. This research establishes that the market level of entry is typically inefficient when an outside-good sector is included.[17] Bagwell and Lee (2023) consider the first-best allocation where the planner chooses both entry and quantity levels under the same setup. They find that, compared to the first-best allocation, the market provides the first-best entry (i.e. $N_E^{FB} = N_E^{mkt} = 1/f_E\,(k+1)$) but too little selection (i.e. c_D^{mkt} is too high); hence, the market provides too many varieties (N^{mkt} is too high) while allocating too little (much) production to low (high) cost realizations. Bagwell and Lee's findings suggest that the market solution provides distortions in selection and quantity levels in the setting we consider in Proposition 1. Hence, in a second-best setting, one might expect that the planner would have an incentive to alleviate the distortions by adjusting the entry level. Surprisingly, however, the market level of entry continues to maximize welfare among all entry levels that induce selection.

We provide an interpretation of this result, represented by (25), in terms of the underlying changes induced by an increase in N_E starting at the market solution. As shown by (12) and (13), the individual terms in the utility function given in (1) can be written as functions of $\lambda \cdot c_D$. In turn, the consumer's welfare $u\,(\alpha, c_D, \lambda)$ given in (14) is also a function of $\lambda \cdot c_D$. Hence, we may interpret (25) by considering the impact of entry on $\lambda \cdot c_D$. Using our comparative statics derivatives as calculated in the Appendix, we find that starting at the market equilibrium, a higher level of entry has opposing effects on c_D

[17] As discussed in the Introduction, this research includes Bagwell and Lee(2018), Bagwell and Lee (2020), Mankiw and Whinston (1986) and Spence (1976). Both Bagwell and Lee (2018) and Bagwell and Lee (2020) study the same second-best problem under quasi-linear preferences; however, the former employs CES preferences while the latter uses MO preferences for the differentiated sectors. Bagwell and Lee (2020) show that entry efficiency depends on the size of α, whereas Bagwell and Lee (2018) find insufficient entry regardless of parameter values. These contrasting findings highlight the importance of how the second sector is modeled.

and λ

$$\frac{dc_D}{dN_E}|_{N_E=N_E^{mkt}} < 0,\ \frac{d\lambda}{dN_E}|_{N_E=N_E^{mkt}} > 0.$$

In fact, we find that these two opposing forces are exactly offsetting

$$\frac{d\lambda \cdot c_D}{dN_E}|_{N_E=N_E^{mkt}} = \lambda\frac{dc_D}{dN_E} + c_D\frac{d\lambda}{dN_E}|_{N_E=N_E^{mkt}} = 0 \tag{27}$$

which implies (25), the zero first-order impact of additional entry at the market solution. Intuitively, the planner can affect the welfare level by adjusting the entry level N_E only insofar as such a change impacts $\lambda \cdot c_D$. But a marginal increase (decrease) in N_E lowers (raises) the selection level c_D and also raises (lowers) the marginal utility of income λ, with the two effects being exactly offseting.[18] Hence, the planner cannot improve welfare with a small change in the level of entry, starting at the market level.

Our findings are related to Demidova's (2017) in that consumer welfare can be expressed as a function of aggregate output, Q.[19] We can rewrite Q as $Q = N \cdot \overline{q}$, where $\overline{q}$ refers to the expected output per variety conditional on survival

$$\overline{q} \equiv E[q(c)|c \leq c_D] = \frac{\lambda \cdot c_D}{2\gamma(k+1)}. \tag{28}$$

Using (10), (27), and the expression for $\overline{q}$ above, we can show that

$$\frac{dN}{dN_E}|_{N_E=N_E^{mkt}} = \frac{d\overline{q}}{dN_E}|_{N_E=N_E^{mkt}} = 0.$$

This implies that additional entry causes offsetting effects on λ and c_D, ensuring that the impact on the extensive margin N and the impact on intensive margin $\overline{q}$ remain unaffected to the first order.

[18] Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2019) encounter a similar knife-edge outcome when firm-level productivity follows a Pareto distribution. In their model, falling trade costs lead to a higher markup by making currently exporting firms charge more, while simultaneously reducing the average markup by attracting less efficient firms. They find that these two effects precisely offset each other under a Pareto distribution. Similarly, we find that this offsetting effect critically depends on the assumption of Pareto distribution. We discuss its role in detail in Section 5.2.

[19] We may confirm this result here as follows. Using (12), we have that $Q = \frac{1}{\eta}(\alpha - \lambda \cdot c_D)$. Referring to (14), we can now easily verify that consumer welfare can be expressed as a function of Q.

3 Two-sector MO model without an outside good: Market outcomes for given entry policies

In this section, we consider the market outcomes in a two-sector MO model without an outside good, when the government may influence the market level of entry by using an entry subsidy or tax. Specifically, we consider the following timeline:

1. For each sector $s \in \{1, 2\}$, the government chooses an entry policy t_{Es}, where $t_{Es} > 0$ ($t_{Es} < 0$) indicates an entry subsidy (tax) in sector s. The total entry subsidy (tax) is levied on (transferred to) consumers in a lump-sum manner.

2. Entry is determined by a Free Entry condition.

3. Entrants pay a fixed cost to observe their respective marginal costs and decide how much to produce, including whether to produce or not. Their respective decisions shape the prices and number of varieties available to consumer.

4. Consumers maximize utility under the given prices and varieties.

In this section, we take the entry policies, t_{Es} for $s \in \{1, 2\}$ as given and determine the resulting market outcomes. We begin our analysis by considering the consumer's problem. We then characterize profit-maximizing behavior by firms and the Free Entry condition.

3.1 Consumer's problem

Just as in the one-sector model, the economy contains a unit mass of identical consumers, each providing a unit of labor in inelastic manner to a competitive labor market, where we now assume as well that there is costless labor mobility across sectors. We normalize the wage as 1. Consumers own symmetric shares of any aggregate net profit or any government transfer, the values of which an individual consumer takes as fixed when choosing consumption.

For the two-sector model, we assume that the consumer's upper-tier utility function is additively separable, so that the consumer maximizes $U_1 + U_2$ where for $s \in \{1, 2\}$

$$U_s = \alpha_s \int_{i \in \Omega_s} q_{is} di - \frac{1}{2}\gamma \int_{i \in \Omega_s} (q_{is})^2 \, di - \frac{1}{2}\eta \left(\int_{i \in \Omega_s} q_{is} di \right)^2 \tag{29}$$

with α_1 possibly different from α_2. The consumer's welfare optimization problem is thus

$$\max_{\{q_{i1}\} \in \Omega_1, \{q_{i2}\} \in \Omega_2} U_1 + U_2$$

s.t.

$$\sum_{s\in\{1,2\}} \int_{i\in\Omega_s} p_{is} q_{is} di = 1 + TR + \sum_{s\in\{1,2\}} \Pi_s \tag{30}$$

where p_{is} and q_{is} are the respective price and quantity of variety i in sector s in the set of available varieties Ω_s in sector s, wage income is normalized as 1, Π_s represents aggregate net profit in sector s and TR refers to the aggregate government transfer. As above, we assume that the preference parameters α, γ and η are all positive.

We consider the Lagrangian

$$L = U_1 + U_2 + \lambda \left(1 + TR + \sum_{s\in\{1,2\}} \Pi_s - \sum_{s\in\{1,2\}} \left(\int_{i\in\Omega_s} p_{is} q_{is} di \right) \right),$$

where $\lambda \geq 0$ is the multiplier for the consumer's optimization problem. Letting $Q_s \equiv \int_{i\in\Omega_s} q_{is} di$ denote aggregate output in sector s, we represent the first-order condition with respect to q_{is} as

$$\alpha_s - \gamma \cdot q_{is} - \eta \cdot Q_s = \lambda p_{is}. \tag{31}$$

As before, integrating (31) over the set of varieties for which $q_{is} > 0$ and letting N_s be the measure of consumed varieties in Ω_s, we obtain

$$\frac{\alpha_s - \lambda \bar{p}_s}{\eta + \frac{\gamma}{N_s}} = Q_s,$$

where $\bar{p}_s$ is the average price of consumed varieties in sector s.

Assuming $\lambda > 0$ and using (31), we see that $p_{is} = \frac{(\alpha_s - \eta \cdot Q_s - \gamma \cdot q_{is})}{\lambda}$ for consumed varieties. We now define $p_s^{\max}$ as the "choke price" for varieties in sector s. We find that

$$p_s^{\max} \equiv \frac{\alpha_s - \eta \cdot Q_s}{\lambda} = \frac{1}{\lambda} \left(\frac{\gamma \cdot \alpha_s + \lambda \cdot \eta \cdot N_s \cdot \bar{p}_s}{\eta \cdot N_s + \gamma} \right). \tag{32}$$

Using (31) and (32), the inverse demand can be written as

$$p_s^d (q_s) = p_s^{\max} - \frac{\gamma}{\lambda} \cdot q_s. \tag{33}$$

3.2 Firm's problem

In sector $s \in \{1, 2\}$, profit maximization for a firm with marginal production cost c gives rise to the profit function

$$\pi_s (c) = \max_q \left(p_s^d (q) - c \right) q$$

For this (two-sector) model of monopolistic competition, the firm takes Q_s, λ and thus the demand intercept $p_s^{\max}$ as given when choosing its profit-maximizing output for sector s. Using (33), we may thus derive the solution to firm's problem as

$$q_s(c) = \frac{\lambda\left(p_s^{\max} - c\right)}{2\gamma}. \tag{34}$$

This solution generates a corresponding profit-maximizing price and profit for the firm:

$$p_s(c) = \frac{p_s^{\max} + c}{2} \tag{35}$$

$$\pi_s(c) = \frac{\lambda}{4\gamma}\left(p_s^{\max} - c\right)^2. \tag{36}$$

A firm in a given sector s produces a positive quantity of its variety provided that its cost realization is no higher than the demand intercept in sector s. A Zero Cutoff Profit (ZCP) condition for sector s thus determines the cost cutoff c_{Ds} as

$$\pi_s(c_{Ds}) = 0$$

or equivalently

$$p_s^{\max} = p_s(c_{Ds}) = c_{Ds},$$

where we assume that $c_{Ds} > 0$.

As in the one-sector model examined above, we assume that costs follow a Pareto distribution, as defined in (9), and are symmetrically distributed across the two sectors. We assume throughout our analysis of the two-sector MO model that $c_{Ds} < c_M$.

Given this distribution, we recall

$$\overline{c}_s \equiv E(c|c \leq c_{Ds}) = \left(\frac{k}{k+1}\right) c_{Ds}.$$

Using (35), $p_s^{\max} = c_{Ds}$ and the Pareto distribution, we find that the average price in sector s can be represented as

$$\overline{p}_s \equiv E(p_s(c)|c \leq c_{Ds}) = \left(\frac{c_{Ds} + \overline{c}_s}{2}\right) = \left(\frac{2k+1}{2(k+1)}\right) c_{Ds}.$$

Notice that the average markup in sector s, $\overline{\mu}_s \equiv \overline{p}_s/\overline{c}_s$, is in fact independent of s and indeed takes the same value as in the one-sector model: for $s \in \{1,2\}$,

$$\overline{\mu}_s = \overline{\mu} = \frac{2k+1}{2k}.$$

Thus, the two-sector model considered here does not admit markup heterogeneity.[20]

Using (32), $p_s^{\max} = c_{Ds}$, and the expression for $\overline{p}_s$ above, we can represent the number of varieties in sector s as

$$N_s = \frac{\gamma \left(\alpha_s - \lambda \cdot p_s^{\max}\right)}{\lambda \cdot \eta \cdot \left(p_s^{\max} - \bar{p}_s\right)} = \frac{2\,(k+1)\,\gamma}{\eta} \frac{\left(\alpha_s - \lambda \cdot c_{Ds}\right)}{\lambda \cdot c_{Ds}}. \tag{37}$$

Similar to the one-sector model, we see from (37) that, once values for c_{Ds} and λ are obtained, the value for the number of available varieties in sector s is determined. For $c_{Ds} > 0$, N_s is strictly decreasing with respect to c_{Ds} for a given value of $\lambda > 0$.

For a given sector s, the number of available varieties can also be represented as a function of the level of entry and the cost cutoff level as $N_s = N_{Es} \cdot G\left(c_{Ds}\right)$. Thus, the value for the number of available varieties in sector s can also be determined given the number of entrants and the cost cutoff level for this sector.

Finally, using (37), $N_s = N_{Es} \cdot G\left(c_{Ds}\right)$ and the Pareto distribution, we further find the relation between N_s and N_{Es} as

$$N_{Es} = \frac{N_s}{G\left(c_{Ds}\right)} = \frac{2\,(k+1)\,\gamma (c_M)^k}{\eta} \frac{\left(\alpha_s - \lambda \cdot c_{Ds}\right)}{\lambda \cdot \left(c_{Ds}\right)^{k+1}}. \tag{38}$$

Hence, we see from (38) that the number of entrants in sector s is determined once values for c_{Ds} and λ are obtained.

3.3 Free Entry Condition

We focus in this section on the policy-induced market outcome; thus, the level of entry is not a direct choice variable but rather is determined for given entry policies by a free entry requirement. Formally, we now impose the Free Entry (FE) condition

$$\int_0^{c_{Ds}} \pi_s\left(c\right) dG\left(c\right) = f_E - t_{Es} \tag{39}$$

where $t_{Es} > 0$ ($t_{Es} < 0$) refers to an entry subsidy (tax) in sector s. The Free Entry condition pins down c_{Ds} for given λ. Specifically, using $\pi_s\left(c\right) = \frac{\lambda}{4\gamma}\left(p_s^{\max} - c\right)^2$, $p_s^{\max} = c_{Ds}$ and the Pareto distribution, we find from (39) that the following relationship between c_{Ds} and λ obtains at the policy-induced market equilibrium:

$$c_{Ds} = \left(\frac{2(k+1)(k+2)\gamma \left(c_M\right)^k \left(f_E - t_{Es}\right)}{\lambda}\right)^{\frac{1}{2+k}} = \lambda^{-\frac{1}{2+k}} \left(f_E - t_{Es}\right)^{\frac{1}{2+k}} \gamma^{\frac{1}{2+k}} \tilde{\phi}^{\frac{1}{2+k}}. \tag{40}$$

[20]By contrast, the difference between $\overline{p}_s$ and $\overline{c}_s$ equals $2c_{Ds}/(k+1)$ and thus varies across sectors to the extent that cost cutoff c_{Ds} does.

where $\tilde{\phi} = 2(k+1)(k+2)(c_M)^k$.

Thus, for a given value of λ, the critical cost cutoff level in sector s, c_{Ds}, is determined by (40). From here, we may determine for sector s the level of entry, the number of varieties available, and the profit-maximizing output and prices. The final step is to use the budget constraint (30) to determine λ. We formally summarize these steps next.

3.4 Equilibrium characterization under fixed entry policies

We now summarize our characterization of the policy-induced market equilibrium outcomes in the two-sector MO model for any sector s, taking as given the entry policies, (t_{E1}, t_{E2}).

1. Using (40), we may determine c_{Ds} for given λ:

$$c_{Ds} = \lambda^{-\frac{1}{2+k}} (f_E - t_{Es})^{\frac{1}{2+k}} \gamma^{\frac{1}{2+k}} \tilde{\phi}^{\frac{1}{2+k}}. \tag{41}$$

2. Using (37) and (38), we then may determine N_s and N_{Es} for given λ :

$$N_s = \frac{2(k+1)\gamma}{\eta} \frac{(\alpha_s - \lambda \cdot c_{Ds})}{\lambda \cdot c_{Ds}} \tag{42}$$

$$N_{Es} = \frac{2(k+1)\gamma (c_M)^k}{\eta} \frac{(\alpha_s - \lambda \cdot c_{Ds})}{\lambda \cdot (c_{Ds})^{k+1}}. \tag{43}$$

3. Using (34) and (35), we may then determine $q_s(c)$ and $p_s(c)$ for given λ :

$$q_s(c) = \frac{\lambda (c_{Ds} - c)}{2\gamma} \tag{44}$$

$$p_s(c) = \frac{c_{Ds} + c}{2} \tag{45}$$

where $p_s^{\max}$ is replaced with c_{Ds} by the ZCP condition.

4. Using (44) and (45), we can then update the budget constraint (30) to determine λ :

$$\sum_{s\in\{1,2\}} N_{Es} \int_0^{c_{Ds}} \frac{\lambda \left((c_{Ds})^2 - c^2\right)}{4\gamma} dG(c) = 1 - \sum_{s\in\{1,2\}} N_{Es} t_{Es} \tag{46}$$

where $\Pi_s = 0$ for $s \in \{1,2\}$ by the Free Entry condition and $TR = -\sum_{s\in\{1,2\}} N_{Es} t_{Es}$.[21]

Hence, for $s \in \{1,2\}$, the policy-induced market equilibrium outcome (c_{Ds}, N_{Es}, N_s, λ, $q_s(c)$, $p_s(c)$) is determined by (41)-(46) for given entry policies, (t_{E1}, t_{E2}).

[21] To confirm that the updated budget constraint (46) follows from the original budget constraint (30),

4 Two-sector MO model without an outside good: Planner's problem

In this section, we consider a planner who chooses N_{E1} and N_{E2} in direct fashion. Thus, we put entry policies to the side in this section; however, in Section 6, we show how entry policies can be used to replicate the planner's entry level choices.

For simplicity, we conduct our normative analysis of the two-sector MO model by considering the benefits to the planner of small changes in entry patterns relative to those that obtain at the (undistorted) market equilibrium. We thus use the market equilibrium as a starting point for comparative statics analyses. We can find the market equilibrium by using our results in Section 3 when $t_{Es} = 0$ for $s \in \{1, 2\}$, and we denote the market equilibrium outcome so determined by $(c_{Ds}^{mkt}, N_{Es}^{mkt}, N_s^{mkt}, \lambda^{mkt}, q_s^{mkt}(c), p_s^{mkt}(c))$.[22] We then allow the planner to move the entry levels slightly away from their market equilibrium levels, leading in turn to changes in endogenous market variables.

Formally, the planner uses consumer welfare to evaluate different values of N_{E1} and N_{E2}. But the planner also faces constraints; namely, (i) a resource constraint derived from the budget constraint (30) with $TR = 0$, (ii) a constraint on the relationship between N_{Es}, c_{Ds} and λ as given in (38), and (iii) a profit-maximizing constraint under which the quantity of variety i consumed is determined by the corresponding firm's cost realization and profit-maximizing output (including zero), as implied by (34) and $p_s^{\max} = c_{Ds}$.

The planner assesses N_{E1} and N_{E2} relative to the objective of maximizing consumer welfare $U \equiv U_1 + U_2$, where for $s \in \{1, 2\}$

$$U_s = \alpha_s \cdot N_{Es} \int_0^{c_{Ds}} q_s(c)\, dG(c) - \frac{\gamma}{2} N_{Es} \int_0^{c_{Ds}} q_s(c)^2\, dG(c) - \frac{\eta}{2} \left(N_{Es} \int_0^{c_{Ds}} q_s(c)\, dG(c) \right)^2. \quad (47)$$

The planner faces a resource constraint

$$\sum_{s \in \{1,2\}} N_{Es} \left(\int_0^{c_{Ds}} c \cdot q_s(c)\, dG(c) + f_E \right) = 1, \quad (48)$$

---

we rewrite (30) as

$$\sum_{s \in \{1,2\}} N_{Es} \int_0^{c_{Ds}} p_s(c) q_s(c) dG(c) = 1 - \sum_{s \in \{1,2\}} N_{Es} t_{Es},$$

where we use that $\Pi_s = 0$ for $s \in \{1, 2\}$ by the Free Entry condition and $TR = -\sum_{s \in \{1,2\}} N_{Es} t_{Es}$. Using (44) and (45), it is now straightforward to confirm (46).

[22] Thus, the market equilibrium outcome obtains from (41)-(46) for the specific case where $t_{Es} = 0$ for $s \in \{1, 2\}$. To reinforce that the market equilibrium is defined for a setting where entry policies are not used, we sometimes refer to the market equilibrium as being "undistorted." Finally, with some abuse of notation, we use λ^{mkt} to represent the market equilibrium value of λ in the two-sector model just as we do above in the one-sector model.

where

$$N_{Es}=\frac{2\left(k+1\right)\gamma\left(c_M\right)^k}{\eta}\frac{\left(\alpha_s-\lambda\cdot c_{Ds}\right)}{\lambda\cdot\left(c_{Ds}\right)^{k+1}}\text{ for }s\in\{1,2\} \tag{49}$$

$$q_s\left(c\right)=\frac{\lambda\left(c_{Ds}-c\right)}{2\gamma}\text{ for }s\in\{1,2\} \tag{50}$$

and where $\lambda\geq 0$ is the multiplier for the consumer's Lagrangian

$$L=U_1+U_2+\lambda\left(1-\sum_{s\in\{1,2\}}\left(N_{Es}\int_{i\in\Omega_s}p_s\left(c\right)q_s\left(c\right)dG\left(c\right)-\Pi_s\right)\right)$$

with $p_s\left(c\right)=\frac{c_{Ds}+c}{2}$.[23]

To interpret this formulation, we note that the consumer utility function represented in (47) follows directly from (29) once profit-maximizing behavior is embedded. We note further that (49) and (50) follow directly from (38) and (34) with $p_s^{\max}=c_{Ds}$, respectively. Finally, to confirm that the resource constraint (48) follows from the budget constraint (30) with $TR=0$, we rewrite the latter as

$$\sum_{s\in\{1,2\}}N_{Es}\int_0^{c_{Ds}}p_s(c)q_s(c)dG(c)=1+\sum_{s\in\{1,2\}}N_{Es}\left(\int_0^{c_{Ds}}\left(p_s(c)-c\right)q_s(c)dG(c)-f_E\right)$$

and simplify.

Following the approach taken in Section 2, we now proceed to rewrite the planner's objective and constraints in terms of α_s, N_{Es}, c_{Ds} and λ. Proceeding as in Section 2 and using (49), we find that U_s from (47) may be rewritten as $u(\alpha_s,c_{Ds},\lambda)$ where the function u is defined in (14). Likewise, it is direct that (49) may be rewritten as $N_{Es}=N_e(\alpha_s,c_{Ds},\lambda)$ where the function N_e is defined in (18). Finally, for the resource constraint (48), we may proceed as in Section 2 while using (49), (50) and the Pareto distribution to rewrite this constraint as $\sum_{s\in\{1,2\}}R\left(\alpha_s,c_{Ds},\lambda\right)=\frac{\eta(2+k)}{\gamma\phi}$, where the function R is defined in (17).

The planner thus evaluates any proposed entry levels N_{E1} and N_{E2} relative to the objective of maximizing

$$\sum_{s\in\{1,2\}}u\left(\alpha_s,c_{Ds},\lambda\right) \tag{51}$$

and with the understanding that c_{D1}, c_{D2} and λ are determined by

$$\sum_{s\in\{1,2\}}R\left(\alpha_s,c_{Ds},\lambda\right)=\frac{\eta\left(2+k\right)}{\gamma\phi} \tag{52}$$

[23]Note also that the value for λ used for our analysis of the two-sector model may differ from that used in the one-sector model.

$$N_{Es} = N_e(\alpha_s, c_{Ds}, \lambda) > 0 \text{ for } s \in \{1, 2\} \tag{53}$$

where $u(\alpha, \lambda, c_D)$, $R(\alpha, \lambda, c_D)$, and $N_e(\alpha, \lambda, c_D)$ are defined in (14), (17), and (18), respectively.

The constraints (52) and (53) represent a 3×3 system with endogenous variables c_{D1}, c_{D2} and λ. We can thus do comparative statics exercises with respect to changes in the exogenous variables, N_{E1} and N_{E2}. With the comparative statics results in place, we can then determine the effects of certain exogenous perturbations on consumer welfare. As mentioned, for the two-sector MO model, we conduct our normative analysis by considering small perturbations around the market equilibrium.

The (undistorted) market equilibrium represented by the vector (N_{E1}^{mkt}, N_{E2}^{mkt}, c_{D1}^{mkt}, c_{D2}^{mkt}, λ^{mkt}) for $s \in \{1, 2\}$ satisfies (52), (53) and the Free Entry condition (40) with $t_{E1} = t_{E2} = 0$ imposed. We assume that the market equilibrium exists and satisfies $N_{Es}^{mkt} > 0$, $c_{Ds}^{mkt} > 0$ and $\lambda^{mkt} > 0$ for $s \in \{1, 2\}$, and we verify the satisfaction of this assumption below.[24] We then appeal to the implicit function theorem to ensure the existence of a solution in c_{D1}, c_{D2} and λ to (52) and (53) for (N_{E1}, N_{E2}) sufficiently close to $(N_{E1}^{mkt}, N_{E2}^{mkt})$. To use this theorem, we require that, at the market solution, the Jacobian determinant associated with (52) and (53) is non-zero.

We consider two kinds of comparative statics exercises. In the first exercise, we consider a symmetric setting in which $\alpha_1 = \alpha_2 \equiv \alpha$ and analyze the implications of a small perturbation in which the planner symmetrically changes N_{E1} and N_{E2} (i.e., $dN_{E1} = dN_{E2}$). This exercise is similar to that analyzed above for the one-sector model. We recall that the change in the entry level for that model induced offsetting changes in λ and c_D. When the two-sector model has a symmetric setting and is subjected to a symmetric change in sectoral entry levels, we find that the results are exactly similar to those in the one-sector model already considered. A second exercise considers a potentially asymmetric setting where α_1 may differ from α_2. For this setting, we allow the planner to consider a small change in N_{E1} and N_{E2} where the change is calibrated so that λ is unaltered (i.e., dN_{E1} and dN_{E2} are such that $d\lambda = 0$). This exercise shares qualitative features with our analysis in Bagwell and Lee (2020) of the welfare effects of an increase in entry into the differentiated sector, where the other sector is an outside-good sector. A unifying feature is that, in both cases, the marginal utility of income remains fixed, and so variety-level consumption is not impacted by changes in the marginal utility of income.

First exercise: For the first exercise, we assume that the setting is symmetric with

[24] As noted previously, with some abuse of notation, we use λ^{mkt} to represent the market equilibrium value for λ in the two-sector model just as we do above in the one-sector model. Below, we distinguish between symmetric settings ($\alpha_1 = \alpha_2$) and potentially asymmetric settings for the two-sector model, and we introduce additional notation as necessary to distinguish (undistorted) market equilibrium variables for these settings from those in the one-sector model.

$\alpha_1 = \alpha_2 \equiv \alpha$, and we analyze the implications of a small perturbation from the market solution in which the planner symmetrically changes N_{E1} and N_{E2} (i.e., $dN_{E1} = dN_{E2}$). Given the symmetry of the setting, the market solution is also symmetric: $N_{E1}^{mkt} = N_{E2}^{mkt}$ and $c_{D1}^{mkt} = c_{D2}^{mkt}$. We can thus simplify the constraint set above and represent it with the following 2×2 system:

$$2 \cdot R(\alpha, c_D, \lambda) = \frac{\eta(2+k)}{\gamma\phi} \tag{54}$$

$$N_E = N_e(\alpha, c_D, \lambda) > 0 \tag{55}$$

with the symmetric solutions for c_D and λ thus determined given a symmetric entry level N_E. For the symmetric setting, the market solution obtains and satisfies these constraints when $N_E = N_{E1}^{mkt} = N_{E2}^{mkt} \equiv \widetilde{N}_E^{mkt}$ and thus $c_D = c_{D1}^{mkt} = c_{D2}^{mkt} \equiv \widetilde{c}_D^{mkt}$ with $\lambda = \widetilde{\lambda}^{mkt}$.[25,26] We note further that for this symmetric setting the market equilibrium relationship (40) with $t_{Es} = 0$ imposed for $s \in \{1, 2\}$ simplifies and takes the form

$$\widetilde{\lambda}^{mkt} = \frac{\gamma\phi}{(\widetilde{c}_D^{mkt})^{2+k}}, \tag{56}$$

which is exactly the same relationship reported in (22) for the one-sector model.

Following the same steps in the one-sector model, we obtain

$$\tilde{N}_E^{mkt} = \frac{1}{2(k+1) f_E} \tag{57}$$

from (54), (55), and (56). As Bagwell and Lee (2023) show in their Appendix E, there exists a unique solution to the system of (54)-(57) satisfying our positive-value assumptions ($\tilde{\lambda}^{mkt} > 0$, $\tilde{c}_D^{mkt} > 0$, $\tilde{N}_E^{mkt} > 0$). Following their logic, we can also show that $\tilde{c}_D^{mkt} < c_M$, so that selection occurs in the market equilibrium, if $\frac{\eta(k+2)}{2(k+1)\alpha} < c_M$ and $f_E > 0$ is sufficiently small. These conditions are satisfied under our maintained restrictions.

In the Appendix, we confirm that the Jacobian determinant for this 2×2 system is negative when evaluated at the market equilibrium ($N_E = \widetilde{N}_E^{mkt}$). Thus, we may apply the implicit function theorem and calculate the response of c_D and λ to a small and symmetric change in entry levels. We find that the derivatives that emerge from this comparative statics exercise take exactly the same form as they did in the one-sector model. Hence, for the symmetric two-sector model, we may conclude that a small and symmetric change in entry levels from their market equilibrium values induces a fall in c_D and a rise in λ.

[25] We use a tilde (˜) as necessary to distinguish definitions relating to (undistorted) market equilibrium variable values in the symmetric setting (first exercise) from those in the one-sector model. Similarly, for the potentially asymmetric setting (second exercise) considered below, we use a hat (ˆ).

[26] For the symmetric two-sector model, a comparison of (57) with (23) confirms that in the market equilibrium the level of entry into any one sector is half the level of entry in the market equilibrium of the one-sector model: $\widetilde{N}_E^{mkt} = (1/2)N_E^{mkt}$.

Given that the derivatives take the same form as in the one-sector model, we can also conclude that the market solution satisfies the planner's first-order condition. We may now summarize our second main finding as follows:

Proposition 2 *Suppose $\alpha_1 = \alpha_2$ and that the planner is restricted to consider only symmetric changes in entry levels in both sectors: $dN_{E1} = dN_{E2} \equiv dN_E$. In this restricted policy space, the symmetric market equilibrium level of entry satisfies the planner's first-order condition, just as in the one-sector model.*

Proof. The proof of Proposition 2 is completed in the Appendix. ■

Second exercise: Turning to the second exercise, we now allow that α_1 may differ from α_2. For this potentially asymmetric setting, we allow the planner to consider a small change in N_{E1} and N_{E2} where the change is calibrated so that λ is unaltered (i.e., dN_{E1} and dN_{E2} are such that $d\lambda = 0$).

We assume that the (undistorted) market equilibrium represented by the vector (N_{E1}^{mkt}, N_{E2}^{mkt}, c_{D1}^{mkt}, c_{D2}^{mkt}, $\widehat{\lambda}^{mkt}$) exists satisfying (52), (53) and the Free Entry condition (40) with $t_{E1} = t_{E2} = 0$ imposed, and at which for $s \in \{1, 2\}$ we have $N_{Es}^{mkt} > 0$, $c_{Ds}^{mkt} > 0$, $\widehat{\lambda}^{mkt} > 0$ and $c_M > c_{Ds}^{mkt}$. Based on our analysis above for the first exercise, we note that these inequalities are all sure to hold if $|\alpha_2 - \alpha_1|$ is sufficiently small.[27] Starting at this solution, the planner imposes a small perturbation to this system, where we now allow the planner to change both N_{E1} and N_{E2} (i.e., $dN_{E1} \neq 0$ and $dN_{E2} \neq 0$) slightly and in a manner that leaves λ unchanged (i.e., $d\lambda = 0$). For a given increase in N_{E1}, we thus must determine the corresponding change in N_{E2} that preserves the value of λ.

We consider the following 3×3 system:

$$\sum_{s \in \{1,2\}} R\left(\alpha_s, c_{Ds}, \lambda\right) = \frac{\eta\left(2 + k\right)}{\gamma \phi} \tag{58}$$

$$N_e\left(\alpha_1, c_{D1}, \lambda\right) - N_{E1} = 0 \tag{59}$$

$$N_e\left(\alpha_2, c_{D2}, \lambda\right) - F(N_{E1}) = 0, \tag{60}$$

where the function F is specified so that, at the market equilibrium, $F(N_{E1}) = N_{E2}$ and

$$F'(N_{E1}) = -\frac{\frac{\partial R(\alpha_1, c_{D1}, \lambda)}{\partial c_D} \frac{\partial N_e(\alpha_2, c_{D2}, \lambda)}{\partial c_D}}{\frac{\partial R(\alpha_2, c_{D2}, \lambda)}{\partial c_D} \frac{\partial N_e(\alpha_1, c_{D1}, \lambda)}{\partial c_D}}. \tag{61}$$

Starting at the market equilibrium, the function F describes the path of the exogenous change in N_{E2} that accompanies a small change in N_{E1}. We note that the market solution

[27]Recall that our assumption that $c_M > c_{Ds}^{mkt}$ utilizes as well our maintained parameter restrictions.

satisfies the constraints given by (58)-(60) when $N_{E1} = N_{E1}^{mkt}$ and thus $c_{Ds} = c_{Ds}^{mkt}$ with $\lambda = \widehat{\lambda}^{mkt}$.

We note further that the market equilibrium relationship (40) with $t_{Es} = 0$ imposed for $s \in \{1, 2\}$ simplifies and takes the form

$$\widehat{\lambda}^{mkt} = \frac{\gamma\phi}{(c_{Ds}^{mkt})^{2+k}} \equiv \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{2+k}}. \tag{62}$$

Thus, even though $\alpha_1 \neq \alpha_2$ is allowed, in the (undistorted) market equilibrium, the cost cutoff level is in fact independent of the sector: $c_{D1}^{mkt} = c_{D2}^{mkt} \equiv \widehat{c}_D^{mkt}$. Using (37) and (38), it follows in turn that, at the market equilibrium, the number of available varieties and the number of entrants are each symmetric across sectors, even though $\alpha_1 \neq \alpha_2$ is allowed. Thus, at the market equilibrium, differences in demand as captured by $\alpha_1 \neq \alpha_2$ do not translate into different market entry patterns across sectors.[28]

In the Appendix, we consider the Jacobian for the 3×3 system described in (58)-(60) when evaluated at the market equilibrium. We are not able to sign this determinant in general, but we can verify that it is non-zero for a tractable special case. In particular, at the market solution when $\alpha_1 = \alpha_2 \equiv \alpha$, we confirm that the determinant is negative. It thus follows that the determinant of the Jacobian is sure to hold at the market solution when the level of asymmetry (i.e., $|\alpha_2 - \alpha_1|$) is sufficiently small. In order to apply the implicit function theorem, we thus assume henceforth that the level of asymmetry is sufficiently small. We emphasize, however, that what we require as a general matter is simply that the determinant of the Jacobian is non-zero when evaluated at the market solution.

Totally differentiating the system described in (58)-(60) with respect to N_{E1}, using $F(N_{E1}^{mkt}) = N_{E2}^{mkt}$ and (61), and evaluating at the market solution, we report in the Appendix expressions for $\frac{d\lambda}{dN_{E1}}$, $\frac{dc_{D1}}{dN_{E1}}$, $\frac{dc_{D2}}{dN_{E1}}$ and $\frac{\partial u(\alpha_s, c_{Ds}, \lambda)}{\partial c_D}$ when evaluated at the market equilibrium (i.e., at $N_{E1} = N_{E1}^{mkt}$). We show there that $\frac{d\lambda}{dN_{E1}} = 0$ at the market equilibrium; hence, the perturbation captured by our specification in (61) indeed ensures that λ is unchanged. We also find that $\frac{dc_{D1}}{dN_{E1}} < 0 < \frac{dc_{D2}}{dN_{E1}}$ at the market equilibrium. It follows that the reallocation of entry from sector 2 to sector 1 results in a lower cost cutoff level in sector 1 and a higher cost cutoff level in sector 2. Finally, we show that $\frac{\partial u(\alpha_s, c_{Ds}, \lambda)}{\partial c_D} < 0$, which parallels our finding for the one-sector model. As expected, an increase in the cost

[28] Recall that the average markup, $\overline{\mu}$, is symmetric across sectors, even away from the (undistorted) market equilibrium (i.e., even when the Free Entry condition is not imposed). Observe also that, at the market equilibrium, $\overline{p}_s = \left(\frac{2k+1}{2(k+1)}\right) \widehat{c}_D^{mkt}$ and $\overline{c}_s = \left(\frac{k}{k+1}\right) \widehat{c}_D^{mkt}$; hence, while $\alpha_1 \neq \alpha_2$ is allowed, the average price, cost and price-cost difference in the market equilibrium are nevertheless independent of the sector. These three values, however, vary across sectors with the cutoff cost level c_{Ds}, when entry levels are moved away from market equilibrium levels as determined by the Free Entry condition. See also footnote 20.

cutoff for a given sector lowers the consumer utility enjoyed in that sector.

With these calculations in hand, we then examine the impact of the proposed shift in entry levels for consumer welfare. We show in the Appendix that

$$\frac{d}{dN_{E1}} \sum_{s\in\{1,2\}} u(\alpha_s, c_{Ds}, \lambda)|_{N_{E1}=N_{E1}^{mkt}} = \sum_{s\in\{1,2\}} \frac{\partial u(\alpha_s, c_{Ds}, \lambda)}{\partial c_D} \cdot \frac{dc_{Ds}}{dN_{E1}}|_{N_{E1}=N_{E1}^{mkt}} \tag{63}$$
$$= \frac{\eta(\gamma\phi)^3(\widehat{c}_D^{mkt})^{1+k}(2+k)(\alpha_2-\alpha_1)}{D},$$

where

$$D \equiv [4\eta(2+k)(k+1)\gamma(c_M)^k(\widehat{c}_D^{mkt})^{3+2k}][\alpha_1 + k(\alpha_1 - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}}][\alpha_2(\widehat{c}_D^{mkt})^{1+k} + k\gamma\phi] > 0,$$

with the inequality following since $\alpha_1 - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{2+k}} = \alpha_1 - \widehat{\lambda}^{mkt} \cdot \widehat{c}_D^{mkt} > 0$ by $N_{E1}^{mkt} > 0$.

Notice that the described shift in entry levels has no effect on welfare in the special case of a symmetric setting, where $\alpha_1 = \alpha_2$. As we can see from (61), in that case, the exercise involves an increase in N_{E1} that induces an equal-sized decrease in N_{E2}. When the setting is symmetric with $\alpha_1 = \alpha_2$, it is intuitive that, starting at the market solution, a small zero-sum reallocation of entry from one sector to the other would have no first-order welfare effect. As (63) confirms, however, when $\alpha_1 \neq \alpha_2$, the planner can gain from modifying the market solution and expanding the level of entry into one market at the cost of less entry in the other, where the adjustment is made so as to keep λ constant. Interestingly, the market provides too much entry into the sector s for which α_s is highest,which is suggestive of a business-stealing externality interpretation.[29]

We may now summarize our third main finding in the following proposition:

Proposition 3 *Allow $\alpha_1 \neq \alpha_2$ with $|\alpha_2 - \alpha_1|$ sufficiently small so that the Jacobian determinant is non-zero and our assumptions of positive values ($\hat{\lambda}^{mkt} > 0$, $\hat{c}_{Ds}^{mkt} > 0$, $\hat{N}_{Es}^{mkt} > 0$) and $c_M > \hat{c}_{Ds}^{mkt}$ for $s \in \{1,2\}$ are sure to hold. Suppose that the planner is restricted to consider only a small increase in entry into sector 1 that is accompanied by a decrease in entry into sector 2 so as to keep the value for λ fixed: $dN_{E1} > 0 > dN_{E2}$ such that $d\lambda = 0$. In this restricted policy space, starting at the market equilibrium, additional entry in sector 1 raises (lowers) (does not change) welfare U if and only if $\alpha_1 < \alpha_2$ ($\alpha_1 > \alpha_2$) ($\alpha_1 = \alpha_2$).*

[29] If we instead assume $\alpha_1 = \alpha_2$ and allow different fixed entry costs for the two sectors, with $f_{E1} \neq f_{E2}$, then we can similarly establish that the market provides excessive entry into the sector with the lowest fixed cost of entry and thus also the sector with the lowest markup. In support of the latter point, we note that if one sector had a lower fixed cost (as captured in (40) by a larger entry subsidy), then that sector would have a lower critical cost cutoff in the market equilibrium; hence, by (35) and $c_{Ds} = p_s^{\max}$, that sector also would have a lower markup at the market equilibrium.

To understand the forces involved, suppose that $\alpha_1 > \alpha_2$ with the difference small. Starting at the market equilibrium, consider a small increase in entry into sector 1 with a corresponding reduction in entry into sector 2 that keeps the value for λ fixed. Due to $\alpha_1 > \alpha_2$, we can show that this perturbation induces (i) a marginal utility gain from a lower value of c_{D1} that is large in magnitude relative to the induced marginal utility loss from a higher value of c_{D2}, but also (ii) a reduction in c_{D1} that is small in magnitude relative to the induced increase in c_{D2}. When $\alpha_1 > \alpha_2$, we then show that the latter effect dominates, so that the overall level of utility falls. In this way, even though average markups do not vary across sectors, when there are demand differences across sectors ($\alpha_1 \neq \alpha_2$), the interactions in the utility function between the corresponding demand parameters and the cutoff cost levels can support welfare-improving interventions.

The finding in Proposition 3 shares qualitative features with that in Bagwell and Lee (2020) of the welfare effects of an increase in entry into the differentiated sector, where the other sector is an outside-good sector and preferences take a quasi-linear form. Bagwell and Lee show that entry into the differentiated sector is too great if the value for α in that sector exceeds a threshold value.[30] In that model, the level of entry does not impact the marginal utility of consumption; similarly, in the second experiment considered here, an increase of the level of entry into one sector is offset by a decrease in the level of the entry into the other sector, so as to ensure that the marginal utility of income λ is unaltered. Thus, in both cases, the marginal utility of income remains fixed, and so variety-level consumption is not impacted by changes in the marginal utility of income. In addition, and as Proposition 3 confirms, the market provides excessive entry into the sector s with the highest value for α_s, a finding which is broadly analogous to Bagwell and Lee's finding regarding excessive entry into the differentiated sector when the value for α in that sector exceeds a threshold value.

To summarize, we consider two kinds of comparative statics exercises in this section. In the first exercise, we consider a symmetric setting in which $\alpha_1 = \alpha_2$ and analyze the implications of a small perturbation in which the planner symmetrically changes N_{E1} and N_{E2} (i.e., $dN_{E1} = dN_{E2}$). Just as in our analysis of the one-sector model, the symmetric change in entry levels induces a change in λ and impacts variety-level consumption through this channel. Indeed, and as Proposition 2 confirms, when the two-sector model has a symmetric setting and is subjected to a symmetric change in sectoral entry levels, the results are exactly similar to those in the one-sector model. In our second exercise, we allow that α_1 may differ from α_2. For this setting, we consider a small perturbation in which the planner increases the level of entry into sector 1 while simultaneously decreasing

[30]Specifically, in Bagwell and Lee (2020), the market generates excessive entry into the differentiated sector when $\alpha > 2 \cdot c_D^m$, where c_D^m is the cutoff cost level for surviving varieties as determined in the market equilibrium.

the level of entry in sector 2 in such a manner as to ensure that λ is unaltered (i.e., dN_{E1} and dN_{E2} are such that $d\lambda = 0$). We show that such an intervention can improve welfare when demand differences are present across sectors ($\alpha_1 \neq \alpha_2$) even though average markups are symmetric across sectors.

5 Discussion

5.1 MO utility as non-additively seperable preferences

MO preferences are non-additive due to the presence of the parameter $\eta > 0$.[31] As we now explain, this parameter plays an important role in the multi-sector setup.

A positive value for η means the existence of a penalty for aggregate consumption in MO preferences as presented by (1) and (29). To see this, observe that, using (32) and (33), we can rewrite the inverse demand in sector s as

$$p_s^d(q_s) = \frac{\alpha_s - \eta \cdot Q_s}{\lambda} - \frac{\gamma}{\lambda} \cdot q_s.$$

When $\eta > 0$, this demand system has an additional aggregator Q_s, which prevents MO preferences from exhibiting "generalized additive separability (GAS)" (Bertoletti and Etro 2022; Pollak 1972). We refer to the term including the additional aggregator, $\eta \cdot Q_s$, as the η penalty. If $\eta > 0$, an increase in Q_s lowers the value of the y-intercept of the inverse demand in sector s under fixed $\lambda > 0$, which corresponds to a penalty for aggregate consumption in sector s.

The η penalty helps us to understand the intuition for Proposition 3 showing too much entry in the high-demand sector (i.e. the high-α sector). In a market solution, the Free Entry condition requires the same levels of productivity cutoffs, $c_D^{mkt} = c_{D1}^{mkt} = c_{D2}^{mkt}$, as shown by (62). By further using the relation $p_s^{\max} = c_{Ds}$ implied by ZCP, we demonstrate that the y-intercepts of the inverse demands in the two sectors are equalized

$$\frac{\alpha_1 - \eta \cdot Q_1^{mkt}}{\lambda^{mkt}} = c_{D1}^{mkt} = c_{D2}^{mkt} = \frac{\alpha_2 - \eta \cdot Q_2^{mkt}}{\lambda^{mkt}}$$

as long as there are positive entrants in the both sectors, $Q_1^{mkt} > 0$ and $Q_2^{mkt} > 0$. This relation shows that the aggregate consumption penalty is higher in the high-α sector:

$$\eta Q_1^{mkt} - \eta Q_2^{mkt} = \alpha_1 - \alpha_2 > 0$$

if and only if $\alpha_1 > \alpha_2$.[32] This result suggests that the market entry level fails to be efficient

[31] If $\eta = 0$, then MO preferences take a directly additive form.

[32] The result may also provide an intuition for the assumption of sufficiently small $|\alpha_1 - \alpha_2|$. If $|\alpha_1 - \alpha_2|$

since the Free Entry condition induces an excessive η penalty for the high-demand sector.

To further develop this intuition, we decompose the first-order condition from the second exercise shown by (63) into the components of the utility function. We plug (12) and (13) into (1) to rewrite

$$u(\alpha_s, c_{Ds}, \lambda) = U_s^\alpha + U_s^\gamma + U_s^\eta$$

where

$$U_s^\alpha \equiv \frac{\alpha_s(\alpha_s - \lambda \cdot c_{Ds})}{\eta} \tag{64}$$

$$U_s^\gamma \equiv -\frac{(\alpha_s - \lambda \cdot c_{Ds})\lambda \cdot c_{Ds}}{2\eta(2+k)} \tag{65}$$

and

$$U_s^\eta \equiv -\frac{(\alpha_s - \lambda \cdot c_{Ds})^2}{2\eta}. \tag{66}$$

We now impose the same perturbation that we did in Proposition 2, determined by $N_{E2} = F(N_{E1})$ and (61) on the individual components defined by (64), (65), and (66). As shown in the appendix, we can evaluate the signs of the impact on individual terms as

$$\frac{d}{dN_E^1}(U_1^\alpha + U_2^\alpha)|_{N_{E1}=N_{E1}^{mkt}} > 0 \tag{67}$$

$$\frac{d}{dN_E^1}(U_1^\gamma + U_2^\gamma)|_{N_{E1}=N_{E1}^{mkt}} > 0 \tag{68}$$

$$\frac{d}{dN_E^1}(U_1^\eta + U_2^\eta)|_{N_{E1}=N_{E1}^{mkt}} < 0 \tag{69}$$

if and only if $\alpha_1 > \alpha_2$. By construction, the sum of (67), (68), and (69) coincides with (63). We find that the sign of (69) agrees with the whole impact shown by (63) while the signs of (67) and (68) disagree. In this way, the impact of the perturbation on the η term drives the result in Proposition 2 of too much entry in the high-demand sector.

Without the η penalty (i.e. with $\eta = 0$), the utility function (1) is additively separable, allowing us to rewrite the y-intercept of the inverse demand as

$$p_s^{\max} = \frac{\alpha_s}{\lambda} = c_{Ds} \tag{70}$$

by using (4) and (8). Due to the absence of the aggregate consumption penalty, additional entry into a sector does not specifically lower the demand for that sector. Consequently,

is too large, then we may not have enough resources to maintain positive consumption levels in both sectors, $Q_1^{mkt} > 0$ and $Q_2^{mkt} > 0$. Consequently, we could encounter a corner solution with $Q_1^{mkt} > 0$ and $Q_2^{mkt} = 0$.

firms never enter a low-α sector, resulting in a corner solution in the market equilibrium. Formally, by applying (36) and (70), we can rewrite the expected profit in sector s as

$$\int_0^{c_{Ds}} \pi_s(c)\, dG(c) = \frac{(c_{Ds})^{k+2} (c_M)^{-k} \lambda}{2(1+k)(2+k)\gamma} = \frac{(\alpha_s)^{k+2} (c_M)^{-k}}{2(1+k)(2+k)\gamma\lambda^{k+1}}.$$

This indicates that the expected profit from sector 1 is always higher than that from sector 2, regardless of the sizes of Q_1 and Q_2. Therefore, the Free Entry condition necessitates that all entrants belong to sector 1. This result is related to Fally (2022), who argues for greater flexibility by having an additional aggregator under Gorman-Pollak preferences. The η penalty, which includes the additional aggregator Q_s, permits non-corner solutions, illustrating the advantage of the added flexibility provided by the additional aggregator.

In the online appendix, we examine another additively separable preference specification and consider CES preferences. Similar to the two-sector model considered here, two symmetric CES subutilities are additively combined with different weight parameters in the upper-tier utility function. The model differs from the present two-sector model in that, following a standard heterogeneous-firm model under CES preferences, selection is achieved due to the presence of a fixed cost of production. Specifically, in addition to the fixed drawing cost $f_E > 0$, firms pay a fixed cost $f > 0$ for positive production. In this modified setup, all entrants still choose to be in one sector, resulting in a corner solution in the market equilibrium. However, this corner solution can occur in either sector, regardless of the sizes of the weight parameters. In the CES demand system, the price index in a sector with no entry is infinite, implying that a marginal entrant would face zero demand in that sector. Consequently, even if all firms are located in the low-demand sector, no firm has an incentive to deviate to the other sector.[33]

It is also interesting that this aggregate η penalty does not play a role in the entry decision of firms in a single-sector setup. Consider the single-sector model we defined in Section 2. When $\eta = 0$, we can write (8) as

$$p^{\max} \equiv \frac{\alpha}{\lambda} = c_D. \tag{71}$$

By plugging (71) into (5), we can confirm that we have a single aggregator λ in the inverse demand

$$p^d(q) = \frac{(\alpha - \gamma \cdot q)}{\lambda}. \tag{72}$$

The profit-maximizing output and profit of a firm with marginal production cost c facing

[33]Bagwell and Lee (2018) examine an entry efficiency problem akin to the one we consider here, with their two sectors comprised of an outside good and a CES differentiated sector. Bagwell and Lee demonstrate that the entry externality, evaluated at the market solution, is positive, implying that the market offers insufficient entry in their quasi-linear setup.

the demand (72) can be written as

$$q(c) = \frac{\alpha - c \cdot \lambda}{2\gamma} \tag{73}$$

and

$$\pi(c) = \frac{(\alpha - c \cdot \lambda)^2}{4\gamma\lambda}. \tag{74}$$

Correspondingly, we use (73) and (74) to rewrite the resource constraint (19) as

$$N_E \left(\int_0^{\alpha/\lambda} \frac{c(\alpha - c \cdot \lambda)}{2\gamma} dG(c) + f_E \right) = 1 \tag{75}$$

and the Free Entry condition (22) as

$$\int_0^{\alpha/\lambda} \frac{(\alpha - c \cdot \lambda)^2}{4\gamma\lambda} dG(c) = f_E. \tag{76}$$

From (75), it is evident that a higher value of N_E leads to an increased value of λ in order to satisfy the budget constraint. Consequently, this higher value of λ reduces the consumption per individual variety, as shown by (73). This suggests that λ functions in a manner similar to that of the η penalty. In our single-sector setup, the η penalty does not have an independent role; thus, we derive the same market entry level without the η term as

$$N_E^{mkt} = \frac{1}{(k+1) f_E}$$

where we can obtain this result by substituting the isolated λ^{mkt} from the Free Entry condition (76) into the budget constraint (75).

5.2 On the Role of the Unbounded Pareto Distribution

Our analysis above assumes that costs follow a Pareto distribution over the range $[0, c_M]$; equivalently, we assume that productivies are distributed according to an unbounded Pareto distribution. As noted by Bagwell and Lee (2023), this assumption is common in monopolistic competition models featuring firm heterogeneity. They characterize the first-best allocation for the single-sector MO model considered here, demonstrating that the entry level in the first-best allocation agrees with the market-determined entry level (23).[34] Bagwell and Lee investigate the role of the unbounded Pareto distribution for

[34]Bertoletti and Etro (2021) also show that the market solution's entry level and the first-best entry level take the same form as in (23) under the unbounded Pareto distribution and GTP preferences. GTP preferences are indirectly additive and belong to the broader GAS family, though the preferences

the results of the first-best and the market outcome levels, confirming that market entry efficiency is sensitive to this assumption. In the current paper, Proposition 1 shows that the market-determined entry level (23) also solves the second-best problem.

We now explore the role of this assumption for our results, with a particular focus on the finding that the market provides the second-best level of entry as shown by Proposition 1. We demonstrate that the efficiency of market entry is sensitive to the assumption of an unbounded Pareto distribution. To illustrate this, we present a simple example using a bounded Pareto distribution.[35] We specifically examine the alternative assumption that the cost distribution is uniform over the interval $[c_L, c_U]$, where $c_U > c_L > 0$ and thus

$$G(c) = \frac{c - c_L}{c_U - c_L} \tag{77}$$

for $c \in [c_L, c_U]$. The uniform distribution with $c_L > 0$ is equivalent to a bounded Pareto distribution with $k = 1$.

In the appendix, we determine the entry level in the market equilibrium solution under the uniform distribution by following the same steps as in Section 2:

$$N_E^{mkt} = \frac{1}{f_E}\left(\frac{c_D^{mkt} - c_L}{c_L + 2c_D^{mkt}}\right). \tag{78}$$

In the appendix, we also rewrite the planner's problem in Section 2.3 under the uniform distribution as follows

$$\max_{N_E} U(N_E, c_D)$$

s.t.

$$N_E = N_e(N_E, c_D) \tag{79}$$

where

$$U(N_E, c_D) \equiv \frac{3(1 - N_E f_E)}{2(c_D + 2c_L)^2}\left(2(c_D + 2c_L)\alpha + \frac{4(c_U - c_L)(1 - N_E f_E)\gamma}{(c_D - c_L)N_E} + 3\eta(1 - N_E f_E)\right)$$

$$N_e(N_E, c_D) \equiv \frac{\left((c_D)^3 + 2(c_L)^3 - 3c_D(c_L)^2\right)N_E\alpha - 12\gamma c_D(c_U - c_L)(1 - N_E f_E)}{3(c_D - c_L)^2(1 - N_E f_E)\eta}.$$

considered in this paper and Bagwell and Lee (2023) lie outside of the GAS family. See Section IV.B of Bagwell and Lee for a discussion of the role of the Pareto distribution under other preferences, including CES and GAS.

[35]See Feenstra (2018) and Melitz and Redding (2015) for other work that utilizes a bounded Pareto distribution for productivities.

We are interested in the first derivative of the planner's decision as

$$\frac{dU}{dN_E}|_{N_E=N_E^{mkt}} = \frac{\partial U}{\partial N_E}|_{N_E=N_E^{mkt}} + \frac{\partial U}{\partial c_D}\frac{dc_D}{dN_E}|_{N_E=N_E^{mkt}} \tag{80}$$

where $\frac{dc_D}{dN_E}$ is from (79) as

$$\frac{dc_D}{dN_E}|_{N_E=N_E^{mkt}} = \frac{1 - \frac{\partial N_e(N_E,c_D)}{\partial N_E}|_{N_E=N_E^{mkt}}}{\frac{\partial N_e(N_E,c_D)}{\partial c_D}|_{N_E=N_E^{mkt}}}.$$

Since the uniform distribution (77) becomes the Pareto distribution with $k = 1$ when $c_L = 0$, it is straightforward to show that (80) is zero when $c_L = 0$. But it is not feasible to analytically sign (80) when $c_L > 0$. We numerically show that (80) is not necessarily zero. Consider the following parameters: $\alpha = 3$, $\gamma = 1$, $\eta = 1$, $f_E = 0.1$, $c_U = 1$, where we allow $c_L > 0$ to vary. Under this specification, we plug (78) into (79) to get

$$\frac{1}{f_E}\left(\frac{c_D^{mkt} - c_L}{c_L + 2c_D^{mkt}}\right) = N_e\left(\frac{1}{f_E}\left(\frac{c_D^{mkt} - c_L}{c_L + 2c_D^{mkt}}\right), c_D^{mkt}\right)$$

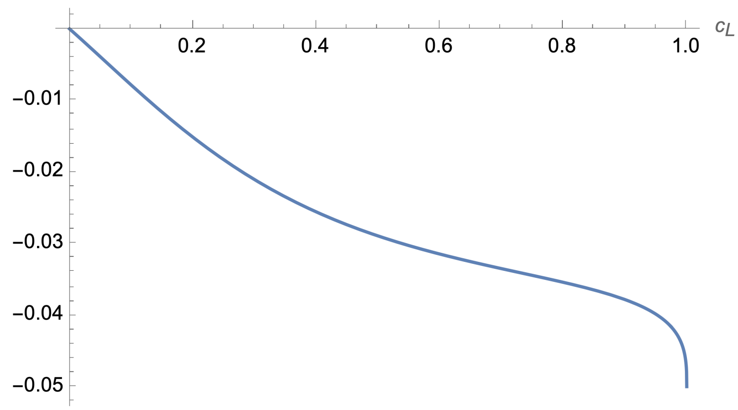
and can numerically solve the above equation for c_D^{mkt}. Then, the solution to the system c_D^{mkt} is a function of c_L, which also induces N_E^{mkt} to be a function of c_L. By plugging these $c_D^{mkt}(c_L)$ and $N_E^{mkt}(c_L)$ functions into (80), we can write (80) as a function of c_L. We plot this numerical relation for $c_L \in [0, c_U = 1]$ in Figure 1. Figure 1a shows the first derivative in (80) is zero only when $c_L = 0$ and shows a negative sign for $c_L > 0$. This implies excessive entry in the market solution. Figure 1b shows $\frac{dU}{dN_E}|_{N_E=N_E^{mkt}}$ for different values of α's for $\alpha \in \{3, 5, 7\}$. All graphs show the same patterns.

In the remainder of the paper, we reimpose the assumption that costs are distributed according to a Pareto distribution over the support $[0, c_M]$.

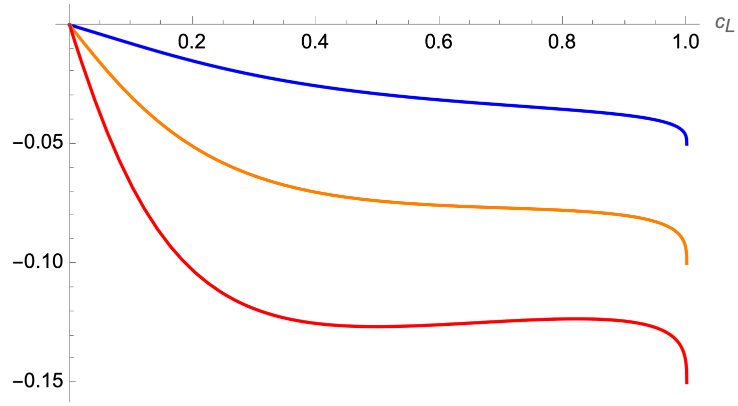
6 Two-sector MO model without an outside good: Replication of planner's problem using entry policies

We show in this section that the market equilibrium outcome generated from the planner's direct choice of entry levels (N_{E1}, N_{E2}) alternatively can be induced by an appropriate choice of entry policies (t_{E1}, t_{E2}) by a government.

To make our argument, we compare two problems. The first problem is the planner's problem, which we define above in (51)-(53). In this problem, the planner directly



(a) $\alpha = 3$



(b) $\alpha = 3$ in **blue**, $\alpha = 5$ in **orange**, $\alpha = 7$ in **red**.

Figure 1: the numerical value of the first derivative of planner's decision for $c \in [0, c_L = 1]$

chooses (N_{E1}, N_{E2}) to maximize aggregate utility $\sum_{s\in\{1,2\}} u(\alpha_s, c_{Ds}, \lambda)$ subject to constraints, where the constraints determine $(c_{D1},\ c_{D2},\ \lambda)$ and thus aggregate utility for a given (N_{E1}, N_{E2}). Below, it will be convenient to represent a candidate choice (N_{E1}, N_{E2}) for the planner and the corresponding values for $(c_{D1},\ c_{D2},\ \lambda)$ as $(N^*_{E1},\ N^*_{E2})$ and $(c^*_{D1},\ c^*_{D2},\ \lambda^*)$, respectively. Thus, given $(N_{E1}, N_{E2}) = (N^*_{E1},\ N^*_{E2})$, the corresponding values $(c^*_{D1},\ c^*_{D2},\ \lambda^*)$ satisfy (52) and (53).

The second problem is *the government's problem.* The government also seeks to maximize aggregate utility $\sum_{s\in\{1,2\}} u(\alpha_s, c_{Ds}, \lambda)$, but the government selects entry policies (t_{E1}, t_{E2}), with the constraints then given by (41)-(46) and corresponding to the market allocations as described in Section 3.4. It will be convenient below to represent a candidate choice $(t_{E1},\ t_{E2})$ for the government and the corresponding values for $(c_{D1},\ c_{D2},\ \lambda)$ as $(t^{**}_{E1},\ t^{**}_{E2})$ and $(c^{**}_{D1},\ c^{**}_{D2},\ \lambda^{**})$, respectively.

Formally, we can represent the constraints for the government's problem as

$$\sum_{s\in\{1,2\}} \frac{2(k+1)\gamma(c_M)^k}{\eta} \frac{(\alpha_s - \lambda\cdot c_{Ds})}{\lambda\cdot(c_{Ds})^{k+1}} \left(\frac{\lambda(c_{Ds})^{k+2}(c_M)^{-k}}{2\gamma(2+k)} + t_{Es}\right) = 1 \qquad (81)$$

$$c_{Ds} = (\lambda)^{-\frac{1}{2+k}} (f_E - t_{Es})^{\frac{1}{2+k}} \gamma^{\frac{1}{2+k}} \tilde{\phi}^{\frac{1}{2+k}} \text{ for } s\in\{1,2\} \qquad (82)$$

where $\tilde{\phi} = 2(k+1)(k+2)(c_M)^k$. Using (43) and the Pareto distribution, we can rewrite (46) as (81). Note that (82) is a restatement of (41). Thus, given $(t_{E1}, t_{E2}) = (t^{**}_{E1}, t^{**}_{E2})$, the corresponding values $(c^{**}_{D1}, c^{**}_{D2},\ \lambda^{**})$ satisfy (81) and (82). Finally, the corresponding values for $(N^{**}_{E1},\ N^{**}_{E2})$ then may be determined using (43).

We now show that any allocation of $(c_{D1}, c_{D2}, \lambda)$ generated by a planner's choice over (N_{E1}, N_{E2}) can be replicated by the government choice of (t_{E1}, t_{E2}), and vice versa. A maintained assumption is that the policies selected by the planner and government are such that the level of entry in each sector is strictly positive: $(N^*_{E1}, N^*_{E2}) > 0$ and $(N^{**}_{E1}, N^{**}_{E2}) > 0$. To state our finding in the simplest possible way, we also assume that for each problem, given the relevant policies, the market equilibrium values for $(c_{D1}, c_{D2}, \lambda)$ are uniquely determined by the corresponding constraints.

Proposition 4 *The proposition has two parts:*
(i). Fix a choice (N^*_{E1}, N^*_{E2}) for the planner and let $(c^*_{D1}, c^*_{D2}, \lambda^*)$ be correspondingly determined by (52) and (53). Then there exists a choice $(t^{**}_{E1}, t^{**}_{E2})$ for the government that correspondingly determines $(c^{**}_{D1}, c^{**}_{D2}, \lambda^{**})$ by (81) and (82) and then $(N^{**}_{E1}, N^{**}_{E2})$ by (43) where $(c^{**}_{D1}, c^{**}_{D2}, \lambda^{**}) = (c^*_{D1}, c^*_{D2}, \lambda^*)$ and $(N^{**}_{E1}, N^{**}_{E2}) = (N^*_{E1}, N^*_{E2})$.
(ii). Fix a choice $(t^{**}_{E1}, t^{**}_{E2})$ for the government that correspondingly determines $(c^{**}_{D1}, c^{**}_{D2}, \lambda^{**})$ by (81) and (82) and then $(N^{**}_{E1}, N^{**}_{E2})$ by (43). Then there exists a choice (N^*_{E1}, N^*_{E2}) for the planner that correspondingly determines $(c^*_{D1}, c^*_{D2}, \lambda^*)$ by (52) and (53) where $(c^*_{D1}, c^*_{D2}, \lambda^*) = (c^{**}_{D1}, c^{**}_{D2}, \lambda^{**})$ and $(N^*_{E1}, N^*_{E2}) = (N^{**}_{E1}, N^{**}_{E2})$.

Proof. The proof of Proposition 4 is found in the Appendix. ■

7 Conclusion

We consider the efficiency of market entry in single- and two-sector closed-economy versions of the Melitz-Ottavanio (MO) model, where differently from the MO model our two-sector model does not involve an outside good. We thereby assess the efficiency of entry under monopolistic competition and heterogeneous firms while allowing for variable markups and general-equilibrium income effects. For each model version, we evaluate whether the market level of entry is efficient relative to the second-best setting in which the planner can control only the level of entry. We show that the market level of entry in the one-sector MO model maximizes welfare among all entry levels that induce selection. For a two-sector MO model without an outside good, we show that the welfare results are exactly similar to those in the one-sector model when the two sectors are symmetric. When the two sectors are asymmetric and the level of asymmetry is sufficiently small, we identify a perturbation indicating a sense in which the market level of entry into the "high-demand" sector is excessive. This intersectoral misallocation occurs at the market equilibrium even though endogenous average markups are equal across sectors. This inefficiency arises due to the presence of an additional aggregator in MO preferences. We also show how the outcomes induced by the planner's direct choice of entry levels alternatively can be induced through the appropriate choice of entry tax/subsidy policies.

8 Appendix

Proof of the two claims in Section 2.3: The first claim is that (12) holds. As noted in the text, the first equality in (12) is true, given profit-maximizing behavior. For the second equality in (12), we use (6), set $p^{\max} = c_D$ and use the Pareto distribution. We find that

$$\int_0^{c_D} q(c)\, dG(c) = \frac{(c_D)^{k+1}(c_M)^{-k}\lambda}{2(1+k)\gamma}.$$

Using this expression and (11), we thus have

$$\begin{aligned} N_E \int_0^{c_D} q(c)\, dG(c) &= \frac{2(k+1)\gamma (c_M)^k}{\eta} \frac{(\alpha - \lambda \cdot c_D)}{\lambda \cdot (c_D)^{k+1}} \frac{(c_D)^{k+1}(c_M)^{-k}\lambda}{2(1+k)\gamma} \\ &= \frac{\alpha - \lambda \cdot c_D}{\eta}, \end{aligned}$$

confirming (12).

The second claim is that (13) holds. As we discuss in the text, the first equality in (13) holds, given profit-maximizing behavior. To establish the second equality in (13), we again use (6), set $p^{\max} = c_D$ and use the Pareto distribution. We find that

$$\int_0^{c_D} q(c)^2\, dG(c) = \frac{(c_D)^{k+2}(c_M)^{-k}\lambda^2}{2(1+k)(2+k)\gamma^2}.$$

Using this expression and (11), we thus have

$$\begin{aligned} N_E \int_0^{c_D} q(c)^2\, dG(c) &= \frac{2(k+1)\gamma (c_M)^k}{\eta} \frac{(\alpha - \lambda \cdot c_D)}{\lambda \cdot (c_D)^{k+1}} \frac{(c_D)^{k+2}(c_M)^{-k}\lambda^2}{2(1+k)(2+k)\gamma^2} \\ &= \frac{1}{\eta} \frac{(\alpha - \lambda \cdot c_D)\lambda c_D}{\gamma(2+k)}, \end{aligned}$$

confirming (13).■

Proof of Proposition 1: As described in the text, we may define the functions $c_D(N_E)$ and $\lambda(N_E)$ as the solutions to (19) and (20), where we know that there exists a unique solution satisfying $c_D(N_E) > 0$ and $\lambda(N_E) > 0$ for $N_E \in (0, 1/f_E)$. We also know that there exists $\widetilde{N}_E \in (0, 1/f_E)$ such that the solution satisfies $c_D(N_E) < c_M$ if and only if $N_E > \widetilde{N}_E$. As indicated in Proposition 1, we consider $N_E > \widetilde{N}_E$. For notational simplicity, we represent $c_D(N_E)$ and $\lambda(N_E)$ below as c_D and λ, respectively, whenever the meaning is clear.

We begin by characterizing the component derivatives in the first-order condition (24). Using (14), we find that

$$\frac{\partial u}{\partial c_D} = -\frac{\lambda\left(\alpha + 2(1+k)\lambda \cdot c_D\right)}{2(2+k)\eta} < 0 \tag{83}$$

$$\frac{\partial u}{\partial \lambda} = -\frac{c_D\left(\alpha + 2(1+k)\lambda \cdot c_D\right)}{2(2+k)\eta} < 0, \tag{84}$$

where the inequalities follow given $c_D > 0$ and $\lambda > 0$.

Using the equation system given by (19) and (20), we may conduct comparative statics to determine $\frac{dc_D}{dN_E}$ and $\frac{d\lambda}{dN_E}$. The first step is to obtain the following characterizations:

$$\frac{\partial R}{\partial c_D} = \left((\alpha - 2\lambda \cdot c_D)\frac{k}{\gamma\phi} - \frac{\alpha(k+1)}{\lambda \cdot (c_D)^{k+2}} + \frac{k}{(c_D)^{k+1}}\right) \tag{85}$$

$$\frac{\partial R}{\partial \lambda} = -\left(\frac{k(c_D)^2}{\gamma\phi} + \frac{\alpha}{\lambda^2 \cdot (c_D)^{k+1}}\right) \tag{86}$$

$$\frac{\partial N_e}{\partial c_D} = -\left(\frac{2(k+1)\gamma(c_M)^k}{\lambda\eta}\right)\left(\frac{\alpha + k(\alpha - \lambda \cdot c_D)}{(c_D)^{k+2}}\right) \tag{87}$$

$$\frac{\partial N_e}{\partial \lambda} = -\left(\frac{2(k+1)\gamma(c_M)^k}{\lambda\eta}\right)\left(\frac{\alpha}{\lambda \cdot (c_D)^{k+1}}\right). \tag{88}$$

We next totally differentiate (19) and (20) with respect to N_E. Using (85)-(88), we find that

$$\frac{dc_D}{dN_E} = -\frac{\eta}{2(k+1)\gamma(c_M)^k k}\frac{(k\lambda^2 \cdot (c_D)^{k+3} + \alpha\gamma\phi)}{(\alpha - \lambda \cdot c_D)(\alpha + k\lambda \cdot c_D)} < 0 \tag{89}$$

$$\frac{d\lambda}{dN_E} = \frac{\eta\lambda}{2(k+1)\gamma(c_M)^k k c_D}\frac{2k\lambda^2 \cdot (c_D)^{k+3} + (1+k)\alpha\gamma\phi - k\gamma\phi\lambda \cdot c_D - k\alpha\lambda \cdot (c_D)^{k+2}}{(\alpha - \lambda \cdot c_D)(\alpha + k\lambda \cdot c_D)}, \tag{90}$$

where the inequality follows from $N_E > 0$ and thus $\alpha - \lambda \cdot c_D > 0$.

Using (83) and (84), we may now represent $\frac{du}{dN_E}$ as follows:

$$\begin{aligned}\frac{du}{dN_E} &= \frac{\partial u}{\partial c_D}\left(\frac{dc_D}{dN_E} + \frac{\frac{\partial u}{\partial \lambda}}{\frac{\partial u}{\partial c_D}}\frac{d\lambda}{dN_E}\right) \\ &= \frac{\partial u}{\partial c_D}\left(\frac{dc_D}{dN_E} + \frac{c_D}{\lambda}\frac{d\lambda}{dN_E}\right)\end{aligned} \tag{91}$$

Given (91), we now use (83), (89) and (90) to calculate that

$$\frac{du}{dN_E} = \frac{\lambda\left(\alpha + 2(k+1)\lambda \cdot c_D\right)}{2(2+k)}\frac{\left(\lambda \cdot (c_D)^{k+2} - \gamma\phi\right)}{2(k+1)\gamma(c_M)^k(\alpha + k\lambda \cdot c_D)}. \tag{92}$$

It follows from (92) that the first-order condition (24) holds if and only if

$$\lambda \cdot (c_D)^{k+2} - \gamma\phi = 0. \tag{93}$$

Recall that $\lambda > 0$ and $c_D > 0$. To characterize this root, we solve for $\frac{\alpha-\lambda\cdot c_D}{\lambda\cdot(c_D)^{k+1}}$ from (19) with (93) imposed, plug the solution into (20) and obtain that

$$N_E = \frac{1}{(k+1)f_E} = N_E^{mkt},$$

where the final equality uses (23). We conclude that the planner's first-order condition (24) is uniquely satisfied at the market solution.

We turn next to the second-order condition. We do not report a general characterization for the sign of the second derivative of $u(\alpha, c_D(N_E), \lambda(N_E))$ with respect to N_E, but we are able to sign the second derivative when it is evaluated at the market solution, which as shown above satisfies (93). To express the second-order condition, we differentiate the first-order condition (92) with respect with N_E while using (89) and (90) to capture $\frac{dc_D}{dN_E}$ and $\frac{d\lambda}{dN_E}$, which are evaluated at the market solution with (93) thus holding. After gathering and simplifying the various terms, we find that

$$\frac{d^2u}{d(N_E)^2}|_{N_E=N_E^{mkt}} = \frac{\lambda^2 \cdot (c_D)^{k+1}\,(\alpha + 2(k+1)\lambda \cdot c_D)}{4(2+k)\gamma(c_M)^k(\alpha + k\lambda \cdot c_D)}\frac{dc_D}{dN_E}|_{N_E=N_E^{mkt}} < 0,$$

where the inequality follows from (89). Thus, the second-order condition (26) holds when evaluated at the solution to the first-order condition, or equivalently at the market solution. This completes the proof of Proposition 1.

We now address two other claims made in the text regarding expressions derived above. First, at the market solution, we may use (93) to show that the numerator of the RHS of (90) reduces to $\eta\lambda[k\lambda \cdot (c_D) + \alpha]\gamma\phi > 0$, ensuring that $\frac{d\lambda}{dN_E} > 0$ at the market solution, as claimed. Second, having simplified (90) in this manner, it is now straightforward to use (89), (90) and (93) to confirm (27). ■

Proof of Proposition 2: We find that, at the market solution, the Jacobian determinant for the 2×2 system given by (54) and (55) takes the following form:

$$\begin{aligned} |\widetilde{J}| &\equiv 2\left(\frac{\partial R}{\partial c_D}\frac{\partial N_E}{\partial \lambda} - \frac{\partial R}{\partial \lambda}\frac{\partial N_E}{\partial c_D}\right)|_{N_E=\widetilde{N}_E^{mkt}} \\ &= -\frac{4(k+1)(c_M)^k k}{\eta\phi\lambda^2 \cdot (c_D)^{k+1}}(\alpha - \lambda \cdot c_D)(\alpha + \lambda k c_D)|_{N_E=\widetilde{N}_E^{mkt}} < 0, \end{aligned}$$

where the inequality follows from (18) and our characterizations of $\widetilde{N}_E^{mkt} > 0$, $\widetilde{c}_D^{mkt} > 0$

and $\widetilde{\lambda}^{mkt} > 0$. We note that, if $|J|$ represents the corresponding determinant of equations (19) and (20) in the one-sector model, then $|\widetilde{J}| = 2|J|$. Given $|\widetilde{J}| < 0$ at the market solution, we may apply the implicit function theorem.

It is straightforward to confirm that the comparative statics derivatives that emerge from (54) and (55) in the symmetric two-sector model take exactly the same form as those that emerge from (19) and (20) in the one-sector model. Formally, if for notational clarity we represent the solution to (54) and (55) as $\widetilde{c}_D(N_E)$ and $\widetilde{\lambda}(N_E)$, then we may easily confirm that

$$\frac{d\widetilde{c}_D}{dN_E} = -\frac{2\frac{\partial R}{\partial \lambda}}{|\widetilde{J}|} = -\frac{\frac{\partial R}{\partial \lambda}}{|J|} = \frac{dc_D}{dN_E} \tag{94}$$

$$\frac{d\widetilde{\lambda}}{dN_E} = \frac{2\frac{\partial R}{\partial c_D}}{|\widetilde{J}|} = \frac{\frac{\partial R}{\partial c_D}}{|J|} = \frac{d\lambda}{dN_E} \tag{95}$$

have already verified for the one-sector model that $\frac{dc_D}{dN_E} < 0 < \frac{d\lambda}{dN_E}$ when evaluated at the market equilibrium.

We can now consider the impact on the planner's objective of a small symmetric change in the level of entry, starting at the market equilibrium. For our first exercise, the planner's welfare change as

$$\frac{d\sum_{s\in\{1,2\}} u\left(\alpha_s, c_{Ds}, \lambda\right)}{dN_E}|_{N_E=\widetilde{N}_E^{mkt}} = 2\left(\frac{\partial u}{\partial c_D}\frac{d\widetilde{c}_D}{dN_E} + \frac{\partial u}{\partial \lambda}\frac{d\widetilde{\lambda}}{dN_E}\right)|_{N_E=\widetilde{N}_E^{mkt}}. \tag{96}$$

Using (94) and (95), we can thus rewrite (96) as

$$\frac{d\sum_{s\in\{1,2\}} u\left(\alpha_s, c_{Ds}, \lambda\right)}{dN_E}|_{N_E=\widetilde{N}_E^{mkt}} = 2\left(\frac{\partial u}{\partial c_D}\frac{dc_D}{dN_E} + \frac{\partial u}{\partial \lambda}\frac{d\lambda}{dN_E}\right)|_{N_E=\widetilde{N}_E^{mkt}} = 0,$$

where the equality follows easily upon using (56), (57) and steps similar to those used in the proof of Proposition 1. We conclude that planner's first-order condition (24) holds at the market solution. This completes the proof of Proposition 2. ■

Proof of Proposition 3: Consider the Jacobian $\widehat{J}$ for the 3×3 system described in (58)-(60) when evaluated at the market equilibrium. We are not able to sign this determinant in general, but we can verify that it is non-zero for a tractable special case. In particular, at the market solution when $\alpha_1 = \alpha_2 \equiv \alpha$, we find that the determinant is strictly negative:

$$|\widehat{J}|_{\alpha_1=\alpha_2\equiv\alpha} = -2k\left(\frac{2(k+1)(c_M)^k}{\eta\gamma\phi^2}\right)^2 (\widehat{c}_D^{mkt})^2(\alpha-\widehat{\lambda}^{mkt}\cdot\widehat{c}_D^{mkt})(\alpha+k(\alpha-\widehat{\lambda}^{mkt}\cdot\widehat{c}_D^{mkt}))(\alpha(\widehat{c}_D^{mkt})^{k+1}+k\gamma\phi) < 0, \tag{97}$$

where the inequality follows from (38) and that $(\hat{N}_{Es}^{mkt}, \widehat{\lambda}^{mkt}, \widehat{c}_D^{mkt})$ converges to $(\widetilde{N}_E^{mkt} > 0, \widetilde{c}_D^{mkt} > 0, \widetilde{\lambda}^{mkt} > 0)$ as α_1 and α_2 approach to the same value of α. Given $|\widehat{J}| < 0$ at the market solution when $\alpha_1 = \alpha_2$, we know that $|\widehat{J}| < 0$ is sure to hold at the market solution when the level of asymmetry (i.e., $|\alpha_2 - \alpha_1|$) is sufficiently small. In order to apply the implicit function theorem, we thus assume henceforth that the level of asymmetry is sufficiently small.

Totally differentiating the system described in (58)-(60) with respect to N_{E1}, using $F(N_{E1}^{mkt}) = N_{E2}^{mkt}$ and (61), and evaluating at the market solution, we find that

$$\begin{aligned}
&\frac{d\lambda}{dN_{E1}}|_{N_{E1}=N_{E1}^{mkt}} = 0 \\
&\frac{dc_{D1}}{dN_{E1}}|_{N_{E1}=N_{E1}^{mkt}} = \frac{1}{\frac{\partial N_e(\alpha_1, c_{D1}, \lambda)}{\partial c_D}}|_{N_{E1}=N_{E1}^{mkt}} \\
&\frac{dc_{D2}}{dN_{E1}}|_{N_{E1}=N_{E1}^{mkt}} = -\frac{\frac{\partial R(\alpha_1, c_{D1}, \lambda)}{\partial c_D}}{\frac{\partial R(\alpha_2, c_{D2}, \lambda)}{\partial c_D}\frac{\partial N_e(\alpha_1, c_{D1}, \lambda)}{\partial c_D}}|_{N_{E1}=N_{E1}^{mkt}}.
\end{aligned}$$

Thus, the perturbation captured by our specification in (61) indeed ensures that λ is unchanged.

Using (17), (18) and imposing the market equilibrium condition (62), we find that, for $s \in \{1, 2\}$,

$$\begin{aligned}
\frac{\partial N_e(\alpha_s, c_{Ds}, \lambda)}{\partial c_D}|_{N_{E1}=N_{E1}^{mkt}} &= -\frac{2(k+1)(c_M)^k}{\eta\phi}[\alpha_s + k(\alpha_s - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}})] < 0 \\
\frac{\partial R(\alpha_s, c_{Ds}, \lambda)}{\partial c_D} &= -\frac{1}{\gamma\phi}[\alpha_s + \frac{k\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}}] < 0,
\end{aligned}$$

where $\alpha_s - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}} = \alpha_s - \widehat{\lambda}^{mkt} \cdot \widehat{c}_D^{mkt} > 0$ by $N_{Es}^{mkt} > 0$. Referring to (61), we can now verify that our second experiment entails an increase in entry into sector 1 that is accompanied by a decrease in entry into sector 2, where the entry adjustments are balanced to keep λ unaltered.

Gathering our findings, we may further report that

$$\frac{dc_{D1}}{dN_{E1}}|_{N_{E1}=N_{E1}^{mkt}} = -\frac{\eta\phi}{2(k+1)(c_M)^k}\frac{1}{[\alpha_1 + k(\alpha_1 - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}})]} < 0 \tag{98}$$

where $\alpha_1 - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}} = \alpha_1 - \widehat{\lambda}^{mkt} \cdot \widehat{c}_D^{mkt} > 0$ by $N_{E1}^{mkt} > 0$ and

$$\frac{dc_{D2}}{dN_{E1}}|_{N_{E1}=N_{E1}^{mkt}} = \frac{\eta\phi}{2(k+1)(c_M)^k}\frac{[\alpha_1 + \frac{k\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}}]}{[\alpha_2 + \frac{k\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}}][\alpha_1 + k(\alpha_1 - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}})]} > 0. \tag{99}$$

Hence, the reallocation of entry from sector 2 to sector 1 results in a lower cost cutoff level in sector 1 and a higher cost cutoff level in sector 2.

To examine the impact of the described shift in entry levels on consumer welfare, we must first determine the impact of a change in the cutoff cost level for a sector on consumer welfare. Using (14) and imposing the market solution condition (62), we find that

$$\frac{\partial u(\alpha_s, c_{Ds}, \lambda)}{\partial c_D}|_{N_{E1}=N_{E1}^{mkt}} = \frac{-\gamma\phi}{2\eta(2+k)(\widehat{c}_D^{mkt})^{3+2k}} \left(\alpha_s(\widehat{c}_D^{mkt})^{1+k} + 2(1+k)\gamma\phi\right) < 0, \quad (100)$$

which parallels the finding (83) for the one-sector model. As expected, an increase in the cost cutoff for a given sector lowers the consumer utility enjoyed in that sector.

We are now prepared to analyze the impact of the proposed shift in entry levels for consumer welfare. Specifically, we seek to evaluate

$$\frac{d}{dN_{E1}} \sum_{s\in\{1,2\}} u(\alpha_s, c_{Ds}, \lambda)|_{N_{E1}=N_{E1}^{mkt}} = \sum_{s\in\{1,2\}} \frac{\partial u(\alpha_s, c_{Ds}, \lambda)}{\partial c_D} \cdot \frac{dc_{Ds}}{dN_{E1}}|_{N_{E1}=N_{E1}^{mkt}}.$$

Using (98), (99) and (100), we calculate

$$\sum_{s\in\{1,2\}} \frac{\partial u(\alpha_s, c_{Ds}, \lambda)}{\partial c_D} \cdot \frac{dc_{Ds}}{dN_{E1}}|_{N_{E1}=N_{E1}^{mkt}} = \frac{\eta(\gamma\phi)^3(\widehat{c}_D^{mkt})^{1+k}(2+k)(\alpha_2-\alpha_1)}{D}, \quad (101)$$

where

$$D \equiv [4\eta(2+k)(k+1)\gamma(c_M)^k(\widehat{c}_D^{mkt})^{3+2k}][\alpha_1 + k(\alpha_1 - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{k+1}}][\alpha_2(\widehat{c}_D^{mkt})^{1+k} + k\gamma\phi] > 0,$$

with the inequality again following since $\alpha_1 - \frac{\gamma\phi}{(\widehat{c}_D^{mkt})^{2+k}} = \alpha_1 - \widehat{\lambda}^{mkt} \cdot \widehat{c}_D^{mkt} > 0$ by $N_{E1}^{mkt} > 0$. This completes the proof of Proposition 3. ■

Proof of the impact of additional entry on individual terms in Section 5.1:

We can evaluate (67) as

$$\frac{d}{dN_{E1}}\left(U_1^\alpha + U_2^\alpha\right)|_{N_{E1}=N_{E1}^{mkt}} \quad (102)$$
$$= \frac{\partial U_1^\alpha}{\partial c_{D1}}\frac{dc_{D1}}{dN_{E1}} + \frac{\partial U_2^\alpha}{\partial c_{D2}}\frac{dc_{D2}}{dN_{E1}}|_{N_{E1}=N_{E1}^{mkt}}$$

where we can evaluate

$$\frac{\partial U_s^\alpha}{\partial c_{Ds}} = -\frac{\alpha_s\lambda}{\eta},$$

and where $\frac{dc_{Ds}}{dN_{Es}}|_{N_{E1}=N_{E1}^{mkt}}$ and $\frac{dc_{D2}}{dN_{E1}}|_{N_{E1}=N_{E1}^{mkt}}$ follow from (98) and (99) respectively. We can sign (102) as

$$\frac{d}{dN_{E1}}\left(U_1^{\alpha}+U_2^{\alpha}\right)|_{N_{E1}=N_{E1}^{mkt}}$$
$$=\frac{k\left(c_M\right)^{-k}\left(\alpha_1-\alpha_2\right)\gamma^2\phi^3}{2c_D^{mkt}\left(1+k\right)\left(\left(c_D^{mkt}\right)^{1+k}\left(1+k\right)\alpha_1-k\cdot\gamma\cdot\phi\right)\left(\left(c_D^{mkt}\right)^{1+k}\alpha_2+k\cdot\gamma\cdot\phi\right)}>0$$

if and only if $\alpha_1>\alpha_2$. The inequality holds by $N_{E1}>0$ (i.e. $\alpha_1-\lambda^{mkt}c_D^{mkt}>0$) and

$$\left(c_D^{mkt}\right)^{1+k}\left(1+k\right)\alpha_1-k\cdot\gamma\cdot\phi$$
$$>k\left(\left(c_D^{mkt}\right)^{1+k}\alpha_1-\gamma\cdot\phi\right)$$
$$=k\left(c_D^{mkt}\right)^{1+k}\left(\alpha_1-c_D^{mkt}\cdot\gamma\cdot\phi\left(c_D^{mkt}\right)^{-(2+k)}\right)=k\left(c_D^{mkt}\right)^{1+k}\left(\alpha_1-\lambda^{mkt}c_D^{mkt}\right)>0.$$

By the same logic, we evaluate the remaining terms as

$$\frac{d}{dN_E^1}\left(U_1^{\gamma}+U_2^{\gamma}\right)|_{N_{E1}=N_{E1}^{mkt}}$$
$$=\frac{\left(c_M\right)^{-k}\left(\alpha_1-\alpha_2\right)\gamma^2\phi^3}{4c_D^{mkt}\left(1+k\right)\left(\left(c_D^{mkt}\right)^{1+k}\left(1+k\right)\alpha_1-k\cdot\gamma\cdot\phi\right)\left(\left(c_D^{mkt}\right)^{1+k}\alpha_2+k\cdot\gamma\cdot\phi\right)}>0$$

$$\frac{d}{dN_E^1}\left(U_1^{\eta}+U_2^{\eta}\right)|_{N_{E1}=N_{E1}^{mkt}}$$
$$=-\frac{\left(c_M\right)^{-k}\left(\alpha_1-\alpha_2\right)\gamma^2\phi^3}{2c_D^{mkt}\left(\left(c_D^{mkt}\right)^{1+k}\left(1+k\right)\alpha_1-k\cdot\gamma\cdot\phi\right)\left(\left(c_D^{mkt}\right)^{1+k}\alpha_2+k\cdot\gamma\cdot\phi\right)}<0$$

where each inequality holds if $\alpha_1>\alpha_2$.■

Proof of Proposition 4: To prove part (i), let us first use (17) and (18) and rewrite planner's constraints (52) and (53) as

$$\sum_{s\in\{1,2\}}\frac{2\left(k+1\right)\gamma\left(c_M\right)^k}{\eta}\frac{\left(\alpha_s-\lambda^*\cdot c_{Ds}^*\right)}{\lambda^*\cdot\left(c_{Ds}^*\right)^{k+1}}\left(\frac{\lambda^*\left(c_{Ds}^*\right)^{k+2}\left(c_M\right)^{-k}k}{2\left(1+k\right)\left(2+k\right)\gamma}+f_E\right)=1 \qquad (103)$$

$$N_{Es}^*=\frac{2\left(k+1\right)\gamma\left(c_M\right)^k}{\eta}\frac{\left(\alpha_s-\lambda^*\cdot c_{Ds}^*\right)}{\lambda^*\cdot\left(c_{Ds}^*\right)^{k+1}} \qquad (104)$$

where $(c_{D1}^*,c_{D2}^*,\lambda^*)$ is determined by (103) and (104) under given (N_{E1}^*,N_{E2}^*). Turning now to the government's problem, we let the government pick t_{Es}^{**} for $s \in \{1,2\}$ such that

$$f_E = t_{Es}^{**} + \frac{(\lambda^*)\left(c_{Ds}^*\right)^{k+2}}{\gamma\tilde{\phi}}. \tag{105}$$

If we plug (105) into (103) and simplify, then we obtain

$$\sum_{s\in\{1,2\}} \frac{2(k+1)\gamma(c_M)^k}{\eta}\frac{(\alpha_s - \lambda^*\cdot c_{Ds}^*)}{\lambda^*\cdot\left(c_{Ds}^*\right)^{k+1}}\left[\frac{\lambda^*\left(c_{Ds}^*\right)^{k+2}(c_M)^{-k}}{2\gamma(k+2)} + t_{Es}^{**}\right] = 1. \tag{106}$$

For given $(t_{E1}^{**}, t_{E2}^{**})$, $(c_{D1}^{**}, c_{D2}^{**}, \lambda^{**})$ is determined by (81) and (82) while $(N_{E1}^{**}, N_{E2}^{**})$ is determined by (43). Comparing (105) with (82) and likewise (106) with (81), we conclude that $(c_{D1}^{**}, c_{D2}^{**}, \lambda^{**}) = (c_{D1}^*, c_{D2}^*, \lambda^*)$. Finally, given this equivalence and comparing (104) with (43), we conclude that $(N_{E1}^{**}, N_{E2}^{**}) = (N_{E1}^*, N_{E2}^*)$.

The proof of part (ii) is similar and therefore omitted. ■

Market Entry and the Second-Best Problem under Unbounded Pareto Distribution: It is convenient to note that

$$\int_0^{c_D} q(c)\,dG(c) = \frac{(c_D - c_L)^2\lambda}{4(c_U - c_L)\gamma} \tag{107}$$

$$\int_0^{c_D} q(c)^2\,dG(c) = \frac{(c_D - c_L)^3\lambda^2}{12(c_U - c_L)\gamma^2} \tag{108}$$

$$\int_0^{c_D} c\cdot q(c)\,dG(c) = \frac{(c_D - c_L)^2(c_D + 2c_L)\lambda}{12(c_U - c_L)\gamma} \tag{109}$$

$$\bar{p} = E(p(c)\,|c < c_D) = \frac{3c_D + c_L}{4} \tag{110}$$

where $q(c)$ and $p(c)$ follow from (6), (7) and (8). Using (109) and (110), we rewrite the resource constraint (16) and the demand for entrants (11), respectively, as

$$N_E\left[\frac{\lambda}{\gamma}\frac{(c_D - c_L)^2(2c_L + c_D)}{12(c_U - c_L)} + f_E\right] = 1 \tag{111}$$

$$N_E = \frac{1}{G(c_D)}\frac{\gamma(\alpha - \lambda\cdot c_D)}{\lambda\cdot\eta(c_D - \bar{p})} = \frac{4\gamma(c_U - c_L)}{\eta}\frac{(\alpha - \lambda c_D)}{\lambda(c_D - c_L)^2}. \tag{112}$$

We use the above relations to determine the entry levels in the market equilibrium solution and the second-best solution under the uniform distribution (77).

The market equilibrium solution under the uniform distribution is determined using the same steps as in Section 2. The variables in the market solution c_D^{mkt}, λ^{mkt}, and N_E^{mkt}

are determined by (111), (112), and the Free Entry condition (21). We rewrite (21) under the uniform distribution as

$$\frac{\lambda^{mkt}}{4\gamma}\frac{\left(c_D^{mkt}-c_L\right)^3}{3\left(c_U-c_L\right)}=f_E. \tag{113}$$

By isolating λ^{mkt} from (113) and plugging it into (111), we can write

$$N_E^{mkt}=\frac{1}{f_E}\left(\frac{c_D^{mkt}-c_L}{c_L+2c_D^{mkt}}\right) \tag{114}$$

which reduces to $N_E^{mkt}=1/\left(2f_E\right)$, consistent with (23) when $k=1$.

For our analysis of the second-best solution, we rewrite U under the uniform distribution using (107) and (108) as

$$U=\alpha N_E\frac{\left(c_D-c_L\right)^2\lambda}{4\left(c_U-c_L\right)\gamma}-\frac{\gamma}{2}N_E\frac{\left(c_D-c_L\right)^3\lambda^2}{12\left(c_U-c_L\right)\gamma^2}-\frac{\eta}{2}\left(N_E\frac{\left(c_D-c_L\right)^2\lambda}{4\left(c_U-c_L\right)\gamma}\right)^2. \tag{115}$$

The planner chooses N_E to maximize welfare U subject to (111) and (112) as in Section 2.3 of the paper. We isolate λ from (111) and plug it into (115) and (112) to rewrite the planner's problem as

$$\max_{N_E} U\left(N_E,c_D\right)$$

s.t.

$$N_E=N_e\left(N_E,c_D\right)$$

where

$$U\left(N_E,c_D\right)\equiv\frac{3\left(1-N_Ef_E\right)}{2\left(c_D+2c_L\right)^2}\left(2\left(c_D+2c_L\right)\alpha+\frac{4\left(c_U-c_L\right)\left(1-N_Ef_E\right)\gamma}{\left(c_D-c_L\right)N_E}+3\eta\left(1-N_Ef_E\right)\right)$$

$$N_e\left(N_E,c_D\right)\equiv\frac{\left(\left(c_D\right)^3+2\left(c_L\right)^3-3c_D\left(c_L\right)^2\right)N_E\alpha-12\gamma c_D\left(c_U-c_L\right)\left(1-N_Ef_E\right)}{3\left(c_D-c_L\right)^2\left(1-N_Ef_E\right)\eta}.$$

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9 Online Appendix - CES case

We characterize the market solutions in a two-sector CES model similar to the two-sector MO model in Section 4. There are two main differences. First, the subutility U_s for $s \in \{1,2\}$ follows the CES form

$$U_s = \left(\int_{i \in \Omega_s} (q_{is})^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}} \tag{116}$$

where the consumer's upper-tier utility function remains additively separable. Thus, the consumer maximizes $\alpha_1 U_1 + \alpha_2 U_2$ given weight parameters $\{\alpha_s\}_{s\in\{1,2\}}$ with $\alpha_1 \neq \alpha_2$. Second, in addition to the fixed entry cost f_E, there is a fixed cost of production f, which is standard in CES models with heterogeneous firms. As discussed below, to prevent the indeterminacy problem when there is no entry in a sector, we assume that a firm cannot charge a price lower than its marginal cost. Otherwise, we follow the same setup as in Section 4.

Consumer's problem We represent the consumer's welfare optimization problem as

$$\max_{\{q_{i1}\}\in\Omega_1, \{q_{i2}\}\in\Omega_2} \alpha_1 U_1 + \alpha_2 U_2 \tag{117}$$

subject to the budget constraint

$$\sum_{s\in\{1,2\}} \int_{i\in\Omega_s} p_{is} q_{is} di = I \tag{118}$$

where consumer's income I is composed of labor income (with the wage normalized to 1) and the sum of net-aggregate profits $I = 1 + \sum_{s\in\{1,2\}} \Pi_s$. We consider the Lagrangian

$$L = \alpha_1 U_1 + \alpha_2 U_2 + \lambda \left(I - \sum_{s\in\{1,2\}} \int_{i\in\Omega_s} p_{is} q_{is} di \right),$$

where $\lambda \geq 0$ is the multiplier for the consumer's optimization problem. The first-order condition w.r.t. q_{is} can be written as

$$\alpha_s \left(\int_{i \in \Omega_s} (q_{is})^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{1}{\sigma-1}} (q_{is})^{-\frac{1}{\sigma}} - \lambda p_{is} = 0. \tag{119}$$

By using (116) and (119), we can write the demand as

$$q_{is} = \frac{(\alpha_s)^{\sigma} U_s}{(\lambda \cdot p_{is})^{\sigma}}. \tag{120}$$

The price index in sector s can be written as

$$P_s \equiv \left(\int_{i \in \Omega_s} (p_{is})^{1-\sigma} \, di \right)^{\frac{1}{1-\sigma}}. \tag{121}$$

By plugging (120) into (116), we can write the following relation between λ and P_s

$$P_s = \frac{\alpha_s}{\lambda}. \tag{122}$$

Using (120) and (121), we can rewrite the budget constraint (118) as

$$(P_1)^{1-\sigma} \left(\frac{\alpha_1}{\lambda} \right)^{\sigma} U_1 + (P_2)^{1-\sigma} \left(\frac{\alpha_2}{\lambda} \right)^{\sigma} U_2 = I.$$

By using (122), we can further simplify the above budget constraint as

$$P_1 U_1 + P_2 U_2 = I. \tag{123}$$

Hence, the consumer's problem is reduced to the following decision

$$\max_{U_1, U_2} \alpha_1 U_1 + \alpha_2 U_2 \tag{124}$$

subject to (123).

Since U_1 and U_2 are perfectly substitutable at the upper-tier utility function, we have a corner solution

$$U_s = \frac{I}{P_s} \text{ and } U_{s'} = 0 \tag{125}$$

if and only if

$$\frac{\alpha_s}{P_s} > \frac{\alpha_{s'}}{P_{s'}} \tag{126}$$

for $s \in \{1, 2\}$ and $s' \neq s$. Under (126), by isolating λ from (122) and plugging it into (120), we complete the demand function for individual variety as

$$q_{is}^d = (p_{is})^{-\sigma} (P_s)^{\sigma-1} I \tag{127}$$
$$q_{is'}^d = 0.$$

Firm's problem In sector s with $U_s > 0$, the profit maximization for a firm with marginal c gives rise to the profit function

$$\pi_s(c) = \max_{p_{is}} (p_{is} - c) \, q_{is}^d - f.$$

We represent the profit-maximizing variables as

$$p_s(c) = \frac{\sigma}{\sigma-1}c \tag{128}$$

$$q_s(c) = \left(\frac{\sigma}{\sigma-1}c\right)^{-\sigma}(P_s)^{\sigma-1} I \tag{129}$$

$$r_s(c) \equiv p_s(c)\, q_s(c) = \left(\frac{\sigma}{\sigma-1}c\right)^{1-\sigma}(P_s)^{\sigma-1} I \tag{130}$$

$$\pi_s(c) = \frac{1}{\sigma}r_s(c) = \frac{1}{\sigma}r_s(c) - f. \tag{131}$$

A firm produces if and only if $c < c_{Ds}$ where the production cost cutoff level c_{Ds} is determined by the Zero Cutoff Profit (ZCP) condition

$$\pi_s(c_{Ds}) = 0. \tag{132}$$

Using c_{Ds} defined in (132), we write the "average" cost of firms in sector s $\tilde{c}_{Ds}$ as

$$\tilde{c}_{Ds} \equiv \left(\frac{\int_0^{c_{Ds}} c^{1-\sigma} dG(s)}{G(c_{Ds})}\right)^{\frac{1}{1-\sigma}}. \tag{133}$$

By applying Pareto distribution $G(c) = (c/c_M)^k$ for $c \in [0, c_M]$ as in the main text, we can write a linear relation between $\tilde{c}_{Ds}$ and c_{Ds}

$$\tilde{c}_{Ds} = \left(\frac{k}{1+k-\sigma}\right)^{\frac{1}{1-\sigma}} c_{Ds} \tag{134}$$

where we assume $1+k-\sigma > 0$ following Chaney (2008).

By using (128) and (134), we can simplify Price index in (121) as a function of $\tilde{c}_{Ds}$

$$P_s = (N_s)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1}\tilde{c}_{Ds} \tag{135}$$

where N_s refers to the number of consumed varieties.

We use (130) and observe that the revenue of a firm with production cost c in sector s can be written as

$$r_s(c) = \left(\frac{c}{c_{Ds}}\right)^{1-\sigma} r_s(c_{Ds}) = \left(\frac{c}{c_{Ds}}\right)^{1-\sigma} \sigma \cdot f \tag{136}$$

where the second equality holds by ZCP as captured by (132), which yields $r_s(c_{Ds}) = \sigma \cdot f$.

The Fee Entry condition is written as

$$\int_0^{c_{Ds}} \pi_s(c)\, dG(c) = f_E.$$

We rewrite this expression using (131) and (136) as

$$\int_0^{c_{Ds}} \left(\left(\frac{c}{c_{Ds}} \right)^{1-\sigma} - 1 \right) f dG(c) = f_E.$$

Using the Pareto distribution, we can simplify the above expression and isolate for c_{Ds} as

$$c_{Ds} = \left(\frac{f_E (1 + k - \sigma)}{(\sigma - 1) f} \right)^{1/k} c_M. \tag{137}$$

In sector s', an entrant faces zero expected profit due to the absence of demand. As a result, there are no entrants in sector s':

$$N_{Es'} = 0.$$

Since there is no entry in this sector, it is difficult to determine firms' decisions over prices and quantities. In order to handle the indeterminacy, we assume that a firm cannot charge less than its marginal cost:

$$p_{is'} \geq c_{is}. \tag{138}$$

Using (138), we can rewrite the price index in sector s' (121) as

$$\begin{aligned} P_{s'} &= \left(\int_{i \in \Omega_{s'}} (p_{is'})^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \geq \left(\int_{i \in \Omega_{s'}} (c_{ij})^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \\ &= \left(N_{E_{s'}} \right)^{\frac{1}{1-\sigma}} \left(\int_{i \in \Omega_{s'}} (c_{ij})^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \end{aligned} \tag{139}$$

By taking the limit of $N_{Es'} = 0$ in (139), we can show $P_{s'}$ approaches 0 since $\sigma > 1$. Consequently, in sector s', we have an infinite price index, $P_{s'} = \infty$. This infinite price of unit utility in sector s' is consistent with the condition for the corner solution (126). Interestingly, (126) and (139) imply that $\alpha_s > \alpha_{s'}$ is not necessary to achieve the corner solution with $U_s > 0$ and $U_{s'} = 0$.

By using $N_{Es'} = 0$, the Free Entry condition, (136) and (137), we can rewrite the budget constraint (118) as

$$N_{Es}\left(\int_0^{c_{Ds}}\left(\frac{c}{c_{Ds}}\right)^{1-\sigma}\sigma\cdot f dG\left(c\right)\right)=1.$$

Upon evaluating the above equation using the Pareto distribution, we can determine the number of entrants as

$$N_{Es}=\frac{\sigma-1}{\sigma\cdot k\cdot f_E}. \tag{140}$$

Therefore, the market solution variables in sector s with $U_s>0$, $\left\{p_s^{mkt}\left(c\right),q_s^{mkt}\left(c\right),c_{Ds}^{mkt},P_s^{mkt},N_{Es}^{mkt}\right\}$, can be determined by (128), (129), (134), (135), (137), and (140). In sector s' with $U_{s'}=0$, we have $N_{Es'}^{mkt}=0$, which implies $P_{s'}^{mkt}=\infty$.

Under the characterized market solution, the welfare of a consumer can be expressed as

$$U^{mkt}=\frac{\alpha_s}{P_s^{mkt}} \tag{141}$$

where the price index P_s^{mkt} can be written as a function of parameters

$$P_s^{mkt}=c_M\cdot f^{\frac{k(1+k-\sigma)}{\sigma-1}}\left(f_E\left(1+k-\sigma\right)\right)^{k+\frac{1-k^2}{\sigma-1}}\left(\sigma-1\right)^{\frac{(1+k)(k-\sigma)}{\sigma-1}}\sigma^{\frac{\sigma}{\sigma-1}}. \tag{142}$$

We have established the corner solution case. It remains to exclude a non-corner solution. A non-corner solution to (124) may only exist if

$$\frac{\alpha_1}{P_1}=\frac{\alpha_2}{P_2}. \tag{143}$$

Under (143), consumers are indifferent across any consumption bundles that satisfy the budget constraint (123). This implies that there exists prices $\{p_{i1}\}_{i\in\Omega_1},\{p_{i2}\}_{i\in\Omega_2}$ that construct P_1 and P_2, such that consumers have the same welfare level from $(U_1,U_2)=(I/P_1,0)$ and $(U_1,U_2)=(0,I/P_2)$. However, this scenario cannot be supported by the Free Entry condition as all entrants would be in sector 1 in the former case and in sector 2 in the latter case. In the former case, the price index in sector 2 is infinite ($P_2=\infty$) while P_1 follows (142); in the latter case, $P_1=\infty$ and P_2 follows (142). Therefore, there are no prices that can support both $(I/P_1,0)$ and $(0,I/P_2)$ as part of the solutions to the consumer's problem.